# Some Empirical Evidence on Demand System and Optimal Commodity Taxation* 

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## 1. Introduction

Many empirical studies of optimal commodity taxation tend to indicate that the optimal tax structures are characterized by non-uniformity. Then the uniform taxation, which is a dominant practice in many countries, may necessarily be entailing large dead weight loss.

However, a few recent studies such as Fukushima[1989, 1991] and Fukushima and Hatta [1989] produced results more favorable for uniformity. They have shown that the optimal structure depends crucially on the value of compensated elasticity of labor supply, and that the losses caused by uniform taxation is far smaller than what is implied by some earlier studies ${ }^{1}$ if the compensated labor supply elasticity

[^0]is within the empirically reasonable range. ${ }^{2}$
Although Fukushima and Hatta's results per se are convincing, we cannot take their policy prescriptions as granted for three reasons. The first is an econometric consideration. In their computation, the estimate of demand system was based on the restrictive functional form which does not allow for complementarity. Also, the sample size is too small to be reliable. The second is the specification of labor supply in their model. In their study, the elasticity of labor supply was not obtained from real data, and labor supply is assumed to be weakly separable from consumption. ${ }^{3}$ The third is that the number of commodity groups are too small for practical purposes.

In this study we estimate a complete demand system with a Japanese data set by employing a flexible functional form (Almost Ideal Demand System of Deaton and Muellbauer[1980]). The data are obtained from 47 cities over the period of 1980 to 90 for ten expenditure groups, and consumption of leisure. Unlike time series data, income, wage rate, and prices are available separately for each observation. Thus we can estimate the demand system without restrictive assumptions on preferences with high accuracy.

Based on the estimate of preference parameters, we simulated the uniform tax and optimal commodity tax equilibria. Then we evaluated and compared the excess burdens of optimal and uniform commodity taxations by calculating compensating variation.

The plan of the paper is as follows. Section 2 presents a brief theory of optimal commodity taxation. Section 3 describes the Almost Ideal Demand System. Section 4 briefly describes the estimation procedure. The data used are explained in Section 5. Estimation results are presented in Section 6. Section 7 offers an evaluation of optimal and uniform tax schemes and the conclusions are stated in Section 8.
for efficiency reason.
${ }^{2}$ Fukushima [1989] pointed out the unreasonably high value of compensated elasticity implicit in Atkinson and Stiglitz [1972]. Also Fukushima [1991] pointed out the same for Harris and MacKinnon [1979], and recalculated the optimal tax rates with a set of parameter values which give more reasonable elasticity estimates.
${ }^{3}$ The separability of labor supply from other commodities is decisively rejected by many studies. See for example Barnett [1979] and Browning and Meghir [1991].

## 2. The Optimal Taxation Problem

### 2.1. The Problem Stated

Let us begin by specifying a well behaved utility function of a person given by

$$
\begin{equation*}
u=u\left(q_{1}, \ldots q_{n}\right) \tag{1}
\end{equation*}
$$

where $q_{i}(i=1,2, \ldots, n-1)$ is the consumption of commodity $i$ and $q_{n}$ is the consumption of leisure. The consumer is assumed to maximize (1) subject to the budget constraint of the form

$$
\begin{equation*}
\sum_{i=1}^{n} p_{i} q_{i}=y, \tag{2}
\end{equation*}
$$

where $p_{i}$ is the consumer's price of good $i$, and $y$ is the total endowment income including the lump sum income, ${ }^{4}$ i.e. we have

$$
\begin{equation*}
y \equiv I+p_{n} L \tag{3}
\end{equation*}
$$

where $I$ is the lump sum income and $L$ is the endowment of leisure. ${ }^{5}$
The solution of the maximization problem is called the Marshallian demand function and it is written as

$$
\begin{equation*}
q_{i}=q_{i}\left(p_{1}, \ldots, p_{n}, y\right) \tag{4}
\end{equation*}
$$

Substitute this into (1), we have an indirect utility function

$$
\begin{equation*}
u=v(p, y), \tag{5}
\end{equation*}
$$

[^1]where $p=\left(p_{1}, \ldots, p_{n}\right)$. If we solve this expression for $y$, we get the expenditure function
\[

$$
\begin{equation*}
y=e(p, u) . \tag{6}
\end{equation*}
$$

\]

The optimal tax rates are obtained by maximizing (5) with respect to $p_{i}$ (for $i=1,2, \ldots, n-1)^{6}$ subject to a tax revenue constraint

$$
\begin{equation*}
\left(p-p^{0}\right)^{\prime} q(p, y)=r, \tag{7}
\end{equation*}
$$

where $r$ is the tax revenue and $p^{0}$ is the producer's price vector.
The solution to the problem is the optimal price vector from which we can obtain the optimal tax rates. The first order condition for the maximization is given by

$$
\begin{equation*}
\frac{\partial v(p, y)}{\partial p}-\lambda\left[q(p, y)+\frac{\partial q(p, y)}{\partial p} p\right]=0 \tag{8}
\end{equation*}
$$

and

$$
\begin{equation*}
\left(p-p^{0}\right)^{\prime} q(p, y)-r=0 \tag{9}
\end{equation*}
$$

Though these equations are highly non-linear in prices, we can find the solution by an iterative method. ${ }^{7}$ In this way, once the expenditure function and the Marshallian demand functions are specified and estimated, we can numerically obtain the optimal tax rates.

### 2.2. Structures of Optimal Commodity Taxation

We assumed that there is no lump sum income allowed in the system so that $I=0$. If this were not the case, the first best optimum is attained if all the required government revenue is collected by the lump sum taxation. In the real world, however, most of the taxes actually employed are not lump sum. And the role of the lump sum tax lies in a standard of comparison.

We also assumed that we can not tax on the consumption of leisure. If we can tax leisure consumption, a uniform taxation on all goods and leisure will produce first best optimal since it is analytically equivalent to taxing endowment of leisure. ${ }^{8}$ However, a tax on leisure consumption is not feasible in real world.

[^2]Once the tax on leisure is announced, consumer can always lie that he consumed less leisure than actual, and he can effectively reduce his tax liabilities.

In the optimal taxation theory, these two assumptions prevent us from attaining the first best optimality and push us to search for the second best solutions. There are many characterizations of the second best equilibrium. The followings help us to form intuitions concerning the magnitude of the tax rate vectors though most of them hold under special assumptions ${ }^{9}$. One purpose of our empirical estimates is to see how good these rules predict the magnitude of the optimal tax vector.

The Samuelson's Basic Rule ${ }^{10}$ If an optimal tax structure is attained, a proportional increase in all tax rates reduces proportionally the compensated demand vector.

This basic rule can be used to explain some special situations in the following lines.

Compensated Inverse Price Elasticity Rule If the cross compensated substitution terms among the commodities are all zero, the optimal tax rate of a commodity should be proportional to the inverse of its own compensated price elasticity of demand.

The intuition behind this is as follows. With cross substitution terms all zero, the only substitution effect of a tax rate increase is to reducing its own demand. At the optimal, according to the Samuelson's basic rule, the proportion of the change must be identical across the commodities. Thus the good with a high own compensated elasticity should be charged with low tax rate.

The homogeneity of demand system implies that the price elasticities of the $i$ th good add up to zero. If the cross elasticity terms are all zero, the own price elasticity is equal to the wage elasticity with sign reversed. Thus the compensated inverse price elasticity rule is restated in the following way.

Compensated Wage Elasticity Rule If the cross compensated price elasticities among the commodities are all zero, then the optimal tax rate of a commodity is inversely related to the compensated wage elasticity of demand for that good.

[^3]The economics of this rule may be explained as follows. We know that a uniform commodity taxation is equivalent to a wage taxation. ${ }^{11}$ A wage taxation distorts the consumption-leisure choice in the direction to encouraging leisure consumption. Thus reducing the tax rate on strong substitute of leisure accompanied by a revenue offsetting increase in other rates would reduce the distortion caused by the uniform taxation. If the compensated cross price elasticities are zero, then the resulting non uniformity causes no additional distortion between the commodities. Thus the optimal tax rates are ordered according to the inverse of the value of the wage elasticity.

When the cross price elasticities are not zero, the situation becomes more complex and we can not find a simple rule to decide the magnitude of the optimal tax rates. Corlett and Hague [1953] used a three-good model (with two commodities and leisure), and showed that starting from a uniform taxation on the two commodities, increasing the tax rate on the commodity more complementary with leisure (i.e. the good with lower wage elasticity) accompanied by a revenue offsetting decrease of the other rate improves the welfare.

The intuition behind is similar to the case for inverse wage elasticity rule. The uniform commodity tax encourages leisure consumption. Thus raising the tax rate on the commodity which is more complementary with leisure (i.e. less substitutable for leisure) accompanied with a revenue offsetting decrease on the other tax rate will reduce the leisure consumption, resulting in a distortion reduction. Thus, Corlett and Hague essentially showed that the inverse compensated wage elasticity rule applies to the case of three goods despite the non-zero cross price terms. ${ }^{12}$

When the number of commodities are increased, the Corlett and Hague rule is no guide to the final optimal tax structure except for one special case of equal

[^4]compensated wage elasticity of all goods. For this case, the optimal structure is uniform. ${ }^{13}$

### 2.3. The Role of Compensated Labor Supply Elasticity

The magnitude of wage elasticity of leisure is crucial in the discussion of optimal tax structure. Whenever we examine the optimal tax structure, it is convenient to begin by assuming initial uniform taxation. This, as we repeated many times, is equivalent to wage taxation. Thus when the wage elasticity of leisure is small, the distortion caused by the uniform commodity taxation is small. In the extreme case, when the leisure is completely inelastic in its own price, the uniform commodity taxation is optimal.

The wage elasticity of leisure is closely related to the wage elasticity of labor supply since the labor endowment minus leisure consumption is the labor supply. Thus we can replace the wage elasticity of leisure mentioned above by the labor supply elasticity. In the real world, we expect that the compensated labor supply elasticity is not too high and this is the reason why we expect that the uniform taxation can perform well against optimal commodity taxation. ${ }^{14}$

## 3. The Almost Ideal Demand System

We use the Almost Ideal Demand System (AI Demand System) to obtain the estimate of the expenditure function and the Marshallian demand functions. The log expenditure function of AI Demand System is given by

$$
\begin{equation*}
\log e(p, v)=\alpha_{0}+\sum_{i} \alpha_{i} \log p_{i}+\frac{1}{2} \sum_{i} \sum_{j} \gamma_{i j}^{*} \log p_{i} \log p_{j}+v \beta_{0} \prod_{i} p_{i}^{\beta_{i}} . \tag{10}
\end{equation*}
$$

The linear homogeneity of the expenditure function with respect to the price vector requires the following constraints

$$
\begin{equation*}
\sum_{i} \alpha_{i}=1, \quad \sum_{i} \beta_{i}=0, \quad \sum_{i} \gamma_{i j}^{*}=\sum_{j} \gamma_{i j}^{*}=0 . \tag{11}
\end{equation*}
$$

[^5]The parameters of AI Demand System are estimated from a set of expenditure share equations of the form

$$
\begin{equation*}
w_{i}=\alpha_{i}+\sum_{j} \gamma_{i j} \log p_{j}+\beta_{i} \log (y / P), \quad i=1, . ., M \tag{12}
\end{equation*}
$$

derived by applying the Shephard's lemma to (10). Here, $w_{i}$ is the $i$ th expenditure share, $y$ is the total expenditure, $M$ is the total number of goods, $P$ is the price index defined by ${ }^{15}$

$$
\begin{equation*}
\log P=\alpha_{0}+\sum_{i} \alpha_{i} \log p_{i}+\frac{1}{2} \sum_{i} \sum_{j} \gamma_{i j} \log p_{i} \log p_{j} \tag{13}
\end{equation*}
$$

where

$$
\begin{equation*}
\gamma_{i j}=\frac{1}{2}\left(\gamma_{i j}^{*}+\gamma_{j i}^{*}\right) \tag{14}
\end{equation*}
$$

Under (11), the adding up constraints and homogeneity of the demand functions corresponding to (12) are all satisfied. Needless to say

$$
\begin{equation*}
\gamma_{i j}=\gamma_{j i} \quad \text { for } i, j=1, . ., M \tag{15}
\end{equation*}
$$

The parameter $\alpha_{0}$ can be interpreted as the subsistence expenditure when all the prices are normalized at one.

The Hicksian substitution matrix is given by

$$
\begin{equation*}
S=\left[S_{i j}\right]=\left[\left\{\gamma_{i j}+\beta_{i} \beta_{j} \log (y / P)-w_{i} \delta_{i j}+w_{i} w_{j}\right\} y /\left(p_{i} p_{j}\right)\right], \tag{16}
\end{equation*}
$$

where $\delta_{i j}=1$ if $i=j$, else it is 0 . The negative semi-definiteness of (16) can be checked by computing the eigenvalues of $S$.

Also, the expenditure elasticities $\eta_{i}$ are given by

$$
\begin{equation*}
\eta_{i}=1+\beta_{i} / w_{i} \tag{17}
\end{equation*}
$$

We treat variations in expenditure patterns due to city and time specific factors and other random factors by introducing an additive disturbance to (12). In the estimation we replace $\log P$ by the proxy, $\log P^{*}=\Sigma_{i=1}^{M} w_{i} \log p_{i}$. The effects of this approximation on the parameter estimates were found to be very small in the previous studies. ${ }^{16}$

[^6]We can estimate the parameters by regressing expenditure shares on the log of prices and the proxy for real income, $y / P^{*}$. The estimates of the Hicksian substitution matrix and its negative semi-definiteness depend on the value of $\alpha_{0}$ through the price index $P$ in (13) as well as on the parameters $\alpha_{i}, \beta_{i}, \gamma_{i j}$, prices, and total expenditure. However, it is not hard to specify a reasonable range of subsistence income by using price data. Thus the "estimates" of price elasticities computed by this convention are conditional on $\alpha_{0}$.

## 4. The Estimation Procedure

In the context of our data the econometric specification of share equations in (12) may be written as

$$
\begin{equation*}
w_{i k t}=\alpha_{i}+\sum_{j} \gamma_{i j} \log P_{j}+\beta_{i} \log (Y / P)+u_{i k t}, \tag{18}
\end{equation*}
$$

for $i=1, \ldots, M, k=1, \ldots, N, t=1, \ldots, T$, where additional subscripts $k$ and $t$ represent region and time period, respectively. $u_{i k t}$ is mean zero disturbance term. Since disturbances in our data may depend on time and region specific factors other than prices and income variables, we assume that the disturbance terms have a variance component type structure. Namely, we write $u_{i k t}$ as

$$
\begin{equation*}
u_{i k t}=\mu_{i k}+\lambda_{i t}+\nu_{i k t}, \tag{19}
\end{equation*}
$$

where, $\mu_{i k}$ is the region specific factor which vary across regions but do not change over time, $\lambda_{i t}$ is the time specific factor which uniformly affects all regions in a given year but changes over time, and $\nu_{i k t}$ is other white noise random factors.

It is well-known in the econometric literature that when these time and regional effects are correlated with the explanatory variables (fixed), usual OLS and GLS estimators will be biased. If that is the case we should correct for the biases by introducing time or region specific dummies. On the other hand, when these factors are uncorrelated with the explanatory variables (random), GLS estimator is unbiased and more efficient than dummy variable estimate. ${ }^{17}$

In this study we assumed a mixed model of time and regional effects that time effects are correlated with the RHS variables (fixed) but regional effects are uncorrelated (random). ${ }^{18}$

[^7]
## 5. The Data

The data on expenditures are obtained from Annual Household Expenditure Survey (HES) (Kakei Chosa Hokoku) which consist of a sample survey of worker's household in 47 prefectural capital cities. The published data are average values, and individual observation values are not available. Each year, a total of approximately 5,400 observations are taken from 47 cities, and $1 / 6$ of the observations are replaced by new samples. The survey covers the following expenditure groups:

1. foods (including eating out) [FOOD]
2. housing (rent, repairs and maintenance, and water) [HOUS]
3. fuel, lights, and water [UTIL]
4. furniture and household utensils (including household durables and domestic services) [FURN]
5. clothes and footwear [CLTH]
6. medical care [MEDI]
7. transportation and communication [TRAN]
8. education [EDUC]
9. reading and recreation [RECR]
10. miscellaneous [MISC]

Since housing expenditure in HES does not include home owners' imputed rent in the housing expenditure, we adjusted the figures from National Survey of Family Income and Expenditures (Shohi Jittai Chosa), conducted in 1979, 84 and 89, by proportions of housing tenure type.

The price indices corresponding to the above 10 expenditure groups are obtained from Consumer Price Index Report (Shohisha Bukka Shisu Nenpo) in time series form $(1985=100)$. To account for regional differences in prices, these indices are adjusted to the regional price difference indices in 1982 and 87 .

[^8]Leisure and wage rates figures for those surveyed in HES are not available, so we had to rely on other sources. Work hours and wage rates are obtained from the annual Wage Census (Chingin Sensasu) conducted by the Japan ministry of labor. The monthly work hours are defined as male full-time workers' average work hours (all industries, all ages) per month including overtime work. Wage rates are obtained by dividing the average monthly salary (including bonus payment) by the average work hours. These figures are also available for each prefecture.

The total monetary endowment is defined as wage rate multiplied by total time endowment per month. We defined time endowment as 16 hours per day, ${ }^{19}$ thus monthly time endowment is 480 hours ( 16 hours per day times 30 days), and leisure per month (LSR) is 480 minus work hours. The descriptive statistics for these variables are summarized in Table 2.

The major difficulty in estimating the joint decision of leisure and commodity demand has been lack of appropriate data. It is easy to see that because in macro time series data we do not have a variation in prices and the wage rate in the given year, then we need a long time series, typically over 40 years. But with such data we cannot control for the effects of possible taste changes in estimation without introducing an apriori assumption on structural change. Also, in crosssection data, the wage rate may vary but prices are common to all the households, and so we cannot estimate the effects of price changes. With a large number of observations and varied prices in our data set such problems can be avoided. Furthermore, by introducing time-specific factors we can control for the effects of taste changes and still have enough degrees of freedom to estimate the parameters of flexible functional forms.

## 6. Estimation Results

We estimated the extended demand system ${ }^{20}$ using the 1979 to 1990 data. When all observations were used for estimation, however, the result showed unrealistically high estimate of expenditure elasticity for housing. This is presumably due to the fact that we used housing expenditure data obtained from interpolation. To avoid perverse effect of interpolation we estimated the system from the three time

[^9]periods 1979, 84 and 89 , and 1980, 85 and 90 (sample size of 141) and obtained very reasonable estimates.

### 6.1. Homogeneity, Symmetry and Negative Semi-Definiteness

Table 3 shows estimate of the parameters $\alpha_{i}, \gamma_{i j}$, and $\beta_{i}$ from the restricted model in which homogeneity and symmetry constraints are imposed. The $\chi^{2}$ test statistic for Lagrangian multiplier test for joint restriction of homogeneity and symmetry is 387.4 , which is well in excess of conventional critical values of $\chi^{2}$ with 65 degrees of freedom. However, this should not be overemphasized since a large number of observations like ours tend to reject any null hypothesis of the standard hypothesis test. The restrictions of parameters should be evaluated in terms of economic significance, rather than on purely statistical ground. ${ }^{21}$

Although statistical test of the negative definiteness of the Hicksian substitution matrix cannot be done in AI Demand System, we can check the negativity by looking at its eigenvalues. We evaluated the substitution matrix at sample mean values of explanatory variables and $\alpha_{0}=12.5$ which corresponds to subsistence endowment of 268,000 Yen (hourly wage rate $=560 \mathrm{Yen}$ ). Out of the 11 eigen values 9 of them are negative (one of them is always zero because of adding up property), and the magnitude of positive eigenvalue is very close to zero (see Note 3 of Table 4). Thus the estimates are consistent with microeconomic demand theory.

### 6.2. Test of Weak Separability

We tested the weak separability of leisure and other commodities by using Goldman and Uzawa method. ${ }^{22}$ And we found that the separability is decisively rejected in both statistical and economic senses. ${ }^{23}$

### 6.3. Price and Expenditure Elasticities

Since it is rather hard to get intuition from the original parameter estimates, we discuss the result based on estimates of price and expenditure elasticities. As

[^10]shown in Section 3, price and expenditure elasticities are dependent on expenditure shares which are also functions of prices and total endowment. Table 4 presents estimates of the compensated price and expenditure elasticities in the extended demand system evaluated at sample mean values of prices and wage rate. ${ }^{24}$

The point estimates of the own price elasticities are all negative as they should be, and they are highly significant. The magnitudes of the own price elasticities are given by Table 5, which shows the lowest own price elasticity is 0.284 of utilities, the highest is 1.053 of clothing.

The cross price elasticities exhibit both substitutability and complementarity. Although substitutability is dominant, a significant complementarities are found between seven pairs of commodity groups. They are, housing-(furnishings, clothes), utilities-(transportation, recreation, miscellaneous), furnishings-education, and transportation-education. Also, leisure is a substitute for all the other commodity groups.

The magnitudes of expenditure elasticities are given by Table 6 , which shows that first four items (food, medical, utilities, and leisure) are necessities, and the last seven items (education, housing, clothing, recreation, furnishings, transportation, and miscellaneous) are luxuries. Needless to say, these classifications of commodity groups and the order of elasticities are in accordance with economic common sense.

### 6.4. Compensated Labor Supply Elasticity

The own price leasticities can be translated to the labour supply elasticity whose value is crucial to compute dead weight losses of commodity taxation. The implied labour supply elasticity is 0.39 which is fairly high, but within the range of reasonable estimates. ${ }^{25}$

One fact that separates our result from many others is that the labor supply (i.e. total time endowment minus consumption of leisure) enters into the utility

[^11]function without separability assumption. This is a desirable feature since the separability has been decisively rejected by our test as well as by recent empirical studies. ${ }^{26}$

## 7. Evaluation of Optimal and Uniform Taxation

### 7.1. Optimal and Uniform Tax Rates

Optimal tax rates are computed by solving (8) and (9) by iterative methods. Table 7 shows the estimates of optimal tax rates, the uniform tax rates, and the compensating variations for three tax revenue requirements, 20,50 and 100 thousand yen from a household when its per hour wage rates are assumed to be $1,000,1,500$ and 2,000 yen. When the wage rate is 1,500 yen, the monthly expenditure is around 250 thousand yen, the closest to the sample mean. From the table, we can see that the optimal tax rates are remarkably close to the uniform rates for all tax revenue requirements, though they are not exactly uniform.

Table 9 shows the ranking of the commodities based on their own compensated price elasticities, by their wage elasticities, and by the optimal tax rates. We notice that the neither the compensated inverse elasticity rule nor the compensated wage elasticity rule can predict the ranking of the optimal tax rates at all. This is due to the presence of both substitutability and complementarity in the substitution matrix. ${ }^{27}$

### 7.2. Welfare Losses

We calculated three equilibria - lump sum, uniform, and optimal commodity taxation equilibria - for a series of fixed tax revenue requirements. Then we computed the compensating variations as a measure of the welfare losses with the following procedure.

Let $(\bar{p}, \bar{u}),\left(p^{*}, u^{*}\right)$, and $\left(p^{u}, u^{u}\right)$ be the equilibrium vectors of prices ( $p$ 's) and corresponding utility level ( $u$ 's) under the lump sum taxation, the optimal taxation, and uniform taxation, respectively. Then the compensating variation

[^12]from the lump sum to the optimal taxation is given by
$$
c v=e(\bar{p}, \bar{u})-e\left(\bar{p}, u^{*}\right) .
$$

The equivalent variation takes the form of

$$
e v=e\left(p^{*}, \bar{u}\right)-e\left(p^{*}, u^{*}\right) .
$$

The CV and the EV from the lump sum to uniform taxation are given in a similar fashion. ${ }^{28}$

Table 7 shows the compensating variations and the equivalent variations. We can see that the losses increase with tax revenue requirements.

Lump Sum vs. Optimal and Uniform Taxation If we compare the lump sum taxation to optimal and uniform taxation, we can immediately see that the values of the welfare losses are small. The largest percentage (as a percentage to GNP) is $6.38 \%$ when the wage rate is 1,000 yen/hour and the revenue requirement is 100,000 yen, which is a bit too large a tax revenue to raise from a commodity taxation as the required uniform tax rate is $79.4 \%$. When the optimal tax rates are computed around the sample mean ( 1,500 yen/hour wage rate and 50,000 yen tax revenue), the welfare losses are $0.54 \%$ to $0.59 \%$. When the revenue requirement is small as 20 thousand yen, the losses are even smaller. The results are robust even when the wage rate is increased.
Uniform vs. Optimal Taxation One of the remarkable result seen on Table 7 is that the minuscule differences of dead weight losses between optimal and uniform taxation. The largest is $0.016 \%$ of GNP when the wage rate is 1,000 yen and revenue required is 100,000 yen. Around the sample mean (wage rate is 1,500 yen and the revenue requirement is 50 thousand yen) the difference of the welfare loss is 1 yen, which results in undetectable rate of $0.000 \%$ of GNP. Thus we can safely state that the uniform commodity taxation is indeed a practical substitute for the optimal commodity taxation as long as the efficiency is at issue. ${ }^{29}$

### 7.3. Welfare Losses When Food is not Taxed

Table 8 shows the optimal tax rates when food is not taxed. It shows that the welfare losses are not so large in terms of $\%$ of GNP. However, a comparison to

[^13]Table 7 clearly shows that welfare losses are a great deal larger when food is not taxed.

## 8. Conclusions

There were two major tasks in our project. The first was to obtain estimate of the Japanese extended demand system, which includes households' leisure, income and commodity choice, and is consistent with the theory of demand. We used pooled time series and regional data for the period of 1980 to 1990, and employed a flexible functional form, AI demand system.

Taking advantage of our data structure we estimated the system by controlling for time specific factors. The result implied definite rejection of weak separability of labor supply and commodity choice, and non-rejection of homogeneity and symmetry restrictions on the demand system.

Based on the demand system parameters, we calculated the expenditure and the price elasticities. Both the expenditure elasticities and the compensated price elasticities are within the range expected in the economic common sense. All the own price elasticities were significantly negative, as they should be, and most of them are significant. Also, we found presence of significant substitutability and complementarity across expenditure groups. As for the negativity of substitution matrix, all but one of the eigen values of the substitution matrix were negative. Thus, the result as a whole showed the consistency with the demand theory.

In our model the labor supply enters into the utility function without separability assumptions. The estimated compensated labor supply elasticity was 0.39 , which is within a reasonable range based on the previously reported studies.

The second task was to evaluate the optimal tax equilibrium in relation to the uniform and the lump sum tax equilibria. To this end, we simulated the equilibria under the various wage rates and tax revenue requirements. We found, among other things, that the deadweight losses of uniform and optimal taxations are quite small. In addition, the optimal rates are strikingly close to uniformity, and that the dead weight losses are very close to each other. We also showed that the inappropriateness of the conventional inverse elasticity rules to predict the optimal tax structure.

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Table 1 Variables

| FOOD | Food incl udi ng eat ing out |
| :--- | :--- |
| HOUS | Housi ng i ncl udi ng i mput ed rent |
| UTI L | El ect ri ci ty, gas, I i ght and wat er char ges |
| FURN | Fur ni t ure and househol d ut ensi I s |
| CLTH | Cl ot hes and foot wear |
| MED | Medi cal care |
| TRAN | Tr ansport at i on and commeni cat i on |
| EDUC | Educat i on |
| RECR | Readi ng and recr eat i on |
| M SC | Ot her livi ng expendi ture |
| LSR | Lei sure ( Nbnt hl y endowment (16x30) mi nus mont hl y <br> work hour s) |

TABLE 2 Summary St atistics

Shar es (\%)
1980
1985
1990

| FOOD | 10. 8 ( | 1. 2) | 9.8 ( | 1. 0) | 8.7 ( | 0.9) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HOUS | 4.0 ( | 0.7) | 4.8 ( | 0.8) | 5.2 ( | 0.9) |
| UTI L | 2.1 ( | 0.4) | 2.3 ( | 0.4) | 1. 9 ( | 0.3) |
| FURN | 1.7 ( | 0.3) | 1.7 ( | 0.3) | 1.5 ( | 0.3) |
| CLTH | 3.2 ( | 0.5) | 2.71 | 0.4) | 2.71 | 0.3) |
| MEDI | 0.9 ( | 0.1) | 0.9 ( | 0.1) | 0.9 ( | 0.1) |
| TRAN | 3.51 | 0.7) | 3.6 ( | 0.7) | 3.7 ( | 0.7) |
| EDUC | 1.4 ( | 0.3) | 1.5 ( | 0.3) | 1. 61 | 0.3) |
| RECR | 3.4 ( | 0.6) | 3.4 ( | 0.5) | 3.4 ( | 0.5) |
| M SC | 11.5 ( | 2. 2) | 10.9 ( | 2. 0) | 10.4 ( | 2. 0) |
| LSR | 57.8 ( | 0.7) | 58.0 ( | 0.7) | 58.0 ( | 0.7) |
| Tot al <br> Expendi ture <br> ( 1000 yen) | 255. 3 ( | 18.7) | 310. 7 ( | 25. 6) | 355.8 ( | 32. 0) |

Pri ces( 1980=100)

| FOOD | 100. 0 ( | 3. 2) | 113.9 | 3. 5) | 120.8 ( | 4. 2) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HOUS | 100. 0 ( | 15.4) | 115. 2 ( | 17.0) | 128. 0 ( | 19.4) |
| UTI L | 100. 0 ( | 8. 5) | 110. 7 ( | 8. 0) | 97.7 ( | 7. 3) |
| FURN | 100. 0 ( | 5. 0) | 108. 5 ( | 4. 1) | 107.5 ( | 3. 8) |
| CLTH | 100. 0 ( | 5.7) | 116.7 | 5.6) | 133.6 ( | 7. 2) |
| MEDI | 100. 0 ( | 3. 2) | 117. 1 ( | 2. 6) | 123.9 ( | 2. 6) |
| TRAN | 100. 0 ( | 2. 8) | 110.9 ( | 2. 4) | 113.0 ( | 2. 7) |
| EDUC | 100. 0 ( | 14.2) | 130.7 ( | 17.6) | 158.9 ( | 22.6) |
| RECR | 100. 0 ( | 4. 4) | 113.9 ( | 5.3) | 123.9 ( | 6.9) |
| M SC | 100. 0 ( | 2. 5) | 114. 1 ( | 2. 8) | 121. 1 ( | 3. 5) |


| Wage Rate ( yen/hour) | 1267.8 ( 158.6) | 1574.8( 204. 8) | 1879. 2( 258.3) |
| :---: | :---: | :---: | :---: |
| Wbrk Hours ( hour s/mont h) | 202.1 ( 4.3) | 201. 6( 3. 2) | 201.9( 2.6$)$ |

Note : St andard devi ations in the par ent heses.
2．$\alpha_{0}$ is set to 12.5 which corresponds to the hourly wage rate being 560 yen and the monthly monetary endowment being 268.3 thousand yen． uә人 puesnoy7


Note：The figures in the parentheses are t－values．The estimates are obtained by excluding leisure from the system by the adding up

| $\begin{aligned} & \left(\dagger \varepsilon^{\prime} \varepsilon L\right) \\ & \star \varepsilon \angle 0^{\circ} 0 \end{aligned}$ | $\begin{aligned} & \hline \text { (I6.G) } \\ & \text { Zع10.0 } \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline\left(L L^{\circ} 0\right) \\ & \varepsilon 000.0 \end{aligned}$ | $\begin{gathered} (06 . t) \\ 181000 \end{gathered}$ | $\begin{aligned} & \hline\left(9 \varepsilon^{\circ} \downarrow-\right) \\ & 6 \varepsilon 00^{\circ}- \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline\left(\varepsilon G^{\prime} \varepsilon\right) \\ & \text { S900'0 }^{2} \end{aligned}$ | $\begin{aligned} & \hline\left(L I^{\prime} t\right) \\ & 6900^{\circ} 0 \end{aligned}$ | $\begin{aligned} & \hline(80 \cdot 9-) \\ & \text { Z9000-0- } \end{aligned}$ | $\begin{aligned} & \hline\left(\varepsilon \varepsilon^{\prime} \downarrow\right) \\ & \nabla \vdash 00^{\circ} 0 \end{aligned}$ | $\begin{gathered} (\mathrm{GE} \varepsilon \mathrm{~L}-) \\ \mathrm{G} 9 力 00^{\circ} \end{gathered}$ | $\begin{aligned} & (\mid \mathrm{P} \wedge-7) \\ & (\mathrm{d} / \lambda) \mathrm{u} \\ & \hline \end{aligned}$ | ！ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & (6 \angle O 1-) \\ & 1 \downarrow \& O^{\circ} 0- \end{aligned}$ | $\left(7 \varepsilon^{\circ} 0\right)$ $90000^{\circ} 0$ $\left(\neg L^{\circ} Z\right)$ $\varepsilon \varepsilon เ 0^{\circ} 0$ | $\begin{aligned} & \hline\left(G E^{\prime} \mathrm{L}\right) \\ & G 100^{\circ} 0 \\ & \left(0 G^{\circ} \mathrm{Z}-\right) \\ & 1900^{\circ} 0^{-} \\ & \left(\mathcal{E O}^{\circ} \mathrm{L}\right) \\ & \varepsilon_{1} 0^{\circ} 0 \end{aligned}$ | $\left(Z 8^{\circ} 0-\right)$ $0 Z 00^{\circ} 0^{-}$ $\left(8 \varepsilon^{\circ} Z\right)$ $180^{\circ} 0$ $\left(G G^{\circ} 0-\right)$ $\varepsilon Z 00^{\circ} 0^{-}$ $\left(9 \varepsilon^{\circ} \varepsilon^{-}\right)$ $9900^{\circ} 0^{-}$ |  |  |  |  |  |  |  |  |
|  | $\begin{aligned} & \hline(Z 96) \\ & Z \perp Z 0^{\circ} 0 \end{aligned}$ | $\begin{aligned} & \hline(08 \angle) \\ & 6 \forall 10^{\circ} 0 \end{aligned}$ | $\begin{aligned} & \hline(86 t) \\ & 1810^{\circ} 0 \end{aligned}$ | $\begin{aligned} & \hline\left(16^{\circ}+1\right) \\ & 1 \varepsilon 10^{\circ} 0 \end{aligned}$ | $\begin{aligned} & \hline\left(Z G^{\circ} Z \mathrm{I}\right) \\ & \angle Z Z O \circ \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline\left(18^{\circ} \mathrm{G}\right) \\ & \mathrm{t} 600^{\circ} 0 \end{aligned}$ | $\begin{aligned} & \hline\left(86^{\circ} 9 Z\right) \\ & \text { 七LZO०0 } \end{aligned}$ | $\begin{aligned} & \hline\left(I 8^{\circ} \varepsilon L\right) \\ & \varepsilon G \vdash 0^{\circ} 0 \end{aligned}$ | $\begin{aligned} & \left.\hline 6 L^{\prime} Z \nabla\right) \\ & 9 \nabla \nabla L^{\circ} 0 \end{aligned}$ | $\begin{array}{r} \text { (Ie^-7) } \\ \text { fsuo } \\ \hline \end{array}$ | $!0$ |
| $\begin{array}{\|cc}  & \text { ss!W } \\ \hline 01 & \\ \hline \end{array}$ | $6$ | $\begin{array}{\|rr}  & \text { onp } \\ 8 & \\ \hline \end{array}$ | $L$ | $9 \quad!$ |  | $\mathrm{m}_{\mathrm{t}}$ |  | $Z^{\text {snoH }}$ | $\mathrm{P}^{\mathrm{pOO}}$ | ＝！ |  |

 Note：The elasticities are evaluated at the sample means of log prices，wage rate and total endowment．The numbers in

| $\begin{aligned} & \left(\begin{array}{l} \left(90^{\circ} G L-\right) \\ \text { G8Z'0- } \end{array}\right. \end{aligned}$ | $\begin{aligned} & (1+\forall 8) \\ & \varepsilon \vdash 0^{\prime} 0 \end{aligned}$ | $\left.{ }^{(00} 0^{\circ} \varepsilon I\right)$ $\mid \downarrow \varepsilon 0^{\circ}$ | $\left\lvert\, \begin{aligned} & (Z t 6) \\ & 81000 \end{aligned}\right.$ | $\left\lvert\, \begin{aligned} & (L+\cdot L) \\ & \varepsilon 0^{\circ} 0 \end{aligned}\right.$ | $\left[\begin{array}{l} \left(6 G^{\circ} 9\right) \\ \angle 00^{\prime} 0 \end{array}\right.$ | $\left\lvert\, \begin{aligned} & \left(\varepsilon G^{\prime}+1\right) \\ & \varepsilon Z 0^{\circ} 0 \end{aligned}\right.$ | $\left[\begin{array}{l} (8+9) \\ 8.0^{\circ} 0 \end{array}\right.$ | $\overline{\left(G \varepsilon^{\prime} G\right)}$ $\angle 000$ | $\begin{aligned} & \left(\angle t^{\prime} 0 Z\right) \\ & 6+0^{\circ} 0 \end{aligned}$ | $\begin{aligned} & \text { (68'GI) } \\ & \text { Z90'0 } \end{aligned}$ | $\begin{gathered} (10 \wedge-7) \\ 187 \text { If } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| （1＊＊） | （L9＇G－） | （ $\dagger$ L $\mathcal{L}$ ） | （ $28.1-$ ） | （60 $)^{\text {）}}$ | （150） | （80\％） | （90＇Z） | （0t＇て－） | （G9＇ı－） | （L8＇0） | （10＾－7） |
| 8てで0 | ャ090－ | ع910 | ャ0＇0－ | 1しで0 | 2000 | 1000 | 6900 | S00－ | 6800－ | $89^{\circ} 0$ | os！w ol |
| （00＇\＆） | （ $\llcorner\llcorner\cdot \varepsilon)$ | （ $88.9-)$ | （Gtil） | （0Z゙0－） | （990） | （ャ¢ $0-$ ） | （IG＇I－） | （G0＇E－） | （عL＇0－） | （GG＇0－） | （1e八－7） |
| 6990 | LZG＇0 | S180－ | ちSO\％ | 970．0－ | 080＇0 | 1200－ | $0010-$ | 9て10－ | 9700－ | L90＇0－ | doey 6 |
| （てヤ・ 6 ） | （28．1－） | （GF＇l） | （88．01－） | （L0＇¢－） | （69＇1－） | （ 8＇$^{\circ} \mathrm{l}$－） | （St＇Z－） | （61＇Z） | （67．9） | （ع¢＇Z） | （10＾－7） |
| 8L90 | L8て＇0－ | 1210 | 26L0－ | 0680－ | LSO－0－ | L110 | OS1．0－ | 980＇0 | $988^{\circ} 0$ | $687^{\circ} 0$ | onp 8 |
| （Lヤ＇L） | （60 $\varepsilon$ ） | （0で0－） | （LO＇${ }^{-}$） | （92＇Z－） | （ع1＇ı－） | （60＇ $1-$ ） | （1ヵ0－） | （61＇Z－） | （ $20 \cdot 1-$ ） | （1E゙1） | （10＾－7） |
| 16 「0 $^{0}$ | OG90 | ャてO－0－ | ¢91．0－ | £\＆¢ $0-$ | 8900－ | 860＇0 | L80 0－ | てくし0－ | GSO＇0－ | 6てで0－ | ued $L$ |
| （6G．9） | （1．0） | （99．0） | （6G＇ı－） | （ع1．1－） | （8て＇$\varepsilon$－） | （ $\mathrm{tc}^{\circ} 0$－） | （1800） | （GL＇0） | （99＊） | （9G ${ }^{\text {l }}$ ） | （10＾－7） |
| $9 \dagger^{\circ} 0$ | 0800 | 2110 | ャ600－ | 2920－ | 62L0－ | ع900－ | 2100 | $\angle 90^{\circ} 0$ | ¢60＇0 | $0 \angle 80$ | ！Pow 9 |
| （¢G＇IL） | （800） | （ち¢＇0－） | （ 8 $^{\circ} \mathrm{I}-$ ） | （60＇1－） | （切＂0－） | （86．とا－） | （080） | （90＇$)^{\text {）}}$ | （ $\downarrow$ ¢ $¢-)$ | （0でも） | （18＾－7） |
| ع9t＇0 | ヶ00＇0 | 9800－ | $190^{\circ}$ | G1to | 070－0－ | £SO．- | L100 | GO1\％ | ト110－ | てカャ゙0 | บ7\％g |
| （8t．9） | （90＇Z） | （LG＇1－） | （Gt＇Z－） | （เレ゚0－） | （1800） | （080） | （6L＇G－） | （11．1） | （00＇E－） | （8でZ） | （10＾－7） |
| てカャ゙0 |  | O1で0－ | เカノ゚ 0 | 2800－ | L00＇0 | 080＇0 | 2680－ | 9 $0^{\circ} 0$ | 891．0－ | G9t＇0 | unn」t |
| （G8＇G） | （0ヵ＇て－） | （S0＇E－） | （61＇Z） | （61＇z－） | （GL＇0） | （90＇E） | （11．1） | （6て＇9－） | （ $70 \cdot \mathrm{G}$ ） | （09．E） | （18＾－7） |
| 8L10 | 192＊0－ | 202\％－ | 1900 | LOZ＇0－ | 6200 | カャレ゚ 0 | 8900 | ャ8で0－ | Lع＇0 | くヵ¢ 0 | İ7 ¢ |
| （Lt＇0Z） | （S9＇1－） | （ $8 L^{\circ} 0-$ ） | （67．9） | （ $20 \cdot 1-$ ） | （99＊） |  | （00＇${ }^{\circ}$－） | （ $70 \times \mathrm{G}$ ） | （97LL－） | （19＇Z） | （18＾－7） |
| 899\％ 0 | L800－ | 8100－ | 8110 | Ot0 $0-$ | 8100 | S900－ | G90 ${ }^{-}$ | 890＇0 | G690－ | 860＇0 | snor 2 |
| （68＇G1） | （L8．0） | （G9＇0－） | （ $\varepsilon$ ¢＇Z） | （IE゙1） | （9G＊） | （0でも） | （8でZ） | （09．$\varepsilon$ ） | （IS＇Z） | （978－） | （180－7） |
| 7980 | 990 0 | \＆ $200-$ | カャo 0 | E800－ | GEO＇0 | 0 O10 | 9 $0^{\circ} 0$ | $\angle 20^{\circ}$ | $670{ }^{\circ}$ | て\＆LO－ | poog $\mathrm{l}=$ ！ |
| $1{ }^{1} 7$ | 0s！${ }^{\text {W }}$ | 102y | OnP］ | UE」 | ！PeW | 4710 | unn」 | 1！${ }^{\text {a }}$ | snoH | poos |  |
| 11 | 01 | 6 | 8 | $L$ | 9 | G | $\dagger$ | $\varepsilon$ | Z | 1 | $=$ ？ |


| \％0＇89 | \％1＊1 | \％${ }^{\circ} \varepsilon$ | \％${ }^{\prime}$＇ | \％9 ${ }^{\text {¢ }}$ ¢ | \％60 | \％6＇z | \％9＇1 | \％ 1 ＇ | \％ $0^{\circ} \mathrm{G}$ | \％6\％ | әлеपS |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| （900＇0） | （090＇0） | （G90＇0） | （821．0） | （801．0） | （L600） | （890＇0） | （Z01．0） | （6ヶ0＇0） | （L90＇0） | （980＇0） | （＇＊＇s） |  |
| 9880 | 七99＊ | L88＇ | て८0＇ | †OS＇ | $89^{\circ} 0$ | †てで | Gてが1 | LOLO | $680{ }^{\circ}$ | 6Z90 | K7，${ }^{\text {Sel］}}$ |  |
| 157 | 0s！ N | 10əy | onp | ued | ！ P WW | 4710 | unn」 | $1!7$ | snoh | pood |  |  |



Table 5 Own Price Elasticity

| Utilities | 0.284 |
| :--- | :--- |
| leisure | 0.285 |
| transportation | 0.533 |
| housing | 0.593 |
| miscellaneous | 0.604 |
| medical | 0.726 |
| food | 0.732 |
| education | 0.792 |
| recreation | 0.815 |
| furnishings | 0.892 |
| clothing | 1.053 |

Table 6 Expenditure Elasticity

| food | 0.529 |
| :--- | :--- |
| medical | 0.580 |
| utilities | 0.707 |
| leisure | 0.886 |
| education | 1.022 |
| housing | 1.089 |
| clothing | 1.224 |
| recreation | 1.387 |
| furnishings | 1.425 |
| transportation | 1.504 |
| miscellaneous | 1.539 |




| $\begin{aligned} & \text { 6LO"0 } \\ & \text { ZIE夫 } \end{aligned}$ | 91000 t9才 | Z00 6夫 | $\begin{gathered} \text { 9ع1"0 } \\ \text { OZヶ夫 } \end{gathered}$ | $\begin{aligned} & \hline 9 Z 00 \\ & 8 L 夫 \\ & \hline \end{aligned}$ | $\begin{gathered} \hline \text { E000 } \\ \text { L1夫 } \end{gathered}$ | $\begin{aligned} & \text { عZ\&० } \\ & \text { عIL夫 } \end{aligned}$ | $\begin{gathered} 0900 \\ 801 . \end{gathered}$ | $\begin{gathered} \angle 000 \\ \text { 七レ夫 } \end{gathered}$ | $\begin{aligned} & \hline \text { (dND fo \%) } \\ & \text { !un-7do ^o } \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| てヤて | 99\％ | 600 | 0ャ＊ | 660 | GI＇0 | $10^{\prime} 11$ | ヤでて |  | （dND\％） |
| 6Z8،6夫 | ともて＇て夫 | てヤE夫 | と10＇も1夫 | 890＇E夫 | 8G7＊ | 880＇97夫 | 016‘沠 | E0L＊ | ！un $\wedge \exists$ |
| GE＇Z | GG＊0 | 80\％ | 6でヤ | 960 | G1\％ | 6L＇01 | 0でて | Z®\％ | （dNO\％） |
| もGG「6夫 | Z81「て夫 | て\＆\＆夫 | ャ99「E1夫 | G66＇Z夫 | 8ヤワ夫 | 七6G「もて夫 | G18＊$\downarrow$ 夫 | 069夫 | $7 \mathrm{do} \wedge \exists$ |
| 98.7 | 190 | 60＇0 | 99＇9 | 60＇1 | 91\％ | G9．91 | 七9 ${ }^{\text {¢ }}$ | G8\％ | （dN〇\％） |
| 06G＇レレ＊ | 61ヵ「て夫 | Z9E夫 | LOL＇L1夫 | Z0ヵ＇と夫 | LLも夫 | ZE6＊LE＊ | 98L｀G夫 | 9ヶL夫 | ！un $\wedge 0$ |
| 8L＇Z | 6G＊0 | 60＇0 | とヤ＇G | L0＇1 | G1\％ | てع＇91 | 69＇Z | †ع＇0 | （dND\％） |
| 8Lで11＊ | GSE＇Z夫 | てヤع夫 | L8でL1夫 | ヤて¢＇と夫 | 997夫 | 6IでLE夫 | 8L9「G夫 | 乙EL夫 | 7 do 10 |
| c．9t | 161 | 6.9 | 9＊1L | 8．92 | ع＇6 | L＇6G1 | 9＇Gt | c゙も1 | mxot！ |
| 9．8t | 661 | でL | G＇tL | $8{ }^{\circ} \mathrm{LZ}$ | 96 | 6＇t91 | 0＇Lt | 6＊ | OSIW |
| G．9t | I＇61 | 6.9 | L＇IL | 6．97 | ع＇6 | 1＊091 | 8．9t | 9＊$\downarrow$ | y07y |
| 8とも | 1．81 | $9 \cdot 9$ | G＊$\angle 9$ | 9．9Z | 68 | 1091 | L＇と | 0＊ | 000ヨ |
| 1＇Lt | －61 | $0 \cdot L$ | c＇ZL | でLZ | G＇6 | 6091 | と＇9t | ぐも | NVY1 |
| 00t | 891 | $1 \cdot 9$ | でZ9 | 6 6Z | 78 | ع＇681 | ナード | $\varepsilon \varepsilon 1$ | IOヨW |
| 9\％b | L＇81 | $8 \cdot 9$ | カ0L | † 92 | で6 | 6\％$\angle 1$ | 6 ＊$\dagger$ | どもレ | H 170 |
| ガレも | ع61 | $0{ }^{\circ} \mathrm{L}$ | 9．EL | でLZ | t6 | －891 | カ $9 \downarrow$ | 9＊ | Nサก |
| でじ | でし1 | て＇9 | 1＇ヶ9 | G＇もて | 98 | 8＇とヤ। | と＇Zも | 9 EL | $7 \amalg \cap$ |
| どカワ | ع81 | 99 | ع．89 | 8GZ | 06 | 8＇IG1 | 0＇カワ | 1ヵも | SnOH |
| 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 00 | 000」 |
| Z680 | $968{ }^{\circ}$ | $86 \varepsilon^{\circ} 0$ | Z88＇0 | $888^{\circ} 0$ | $068{ }^{\circ}$ | 七98\％0 | S $\angle 8{ }^{\circ} 0$ | $6 \angle \varepsilon^{\circ} 0$ |  |
| 000＇001夫 | 000＊09夫 | 000＊07夫 | 000＇001夫 | 000＇09夫 | 000＊0Z夫 | 000＂001夫 | 000＇09＊ | 000＇02夫 |  |
| 000＇Z夫 |  |  | 009＇l＊ |  |  | 000＇1＊ |  |  |  |



Table 9 Ranking by Elasticities and Optimal Tax Rates

| By compensated <br> own price elasticity |  | By compensated <br> wage elasticity |  | By optimal <br> tax rate(\%) |  |
| :--- | :---: | :--- | ---: | :--- | ---: |
| util | 0.284 | util | 0.178 | food | 18.2 |
| trans | 0.533 | misc | 0.228 | medi | 18.3 |
| hous | 0.593 | food | 0.364 | util | 18.5 |
| misc | 0.604 | furn | 0.442 | educ | 18.9 |
| medi | 0.726 | dth | 0.463 | hous | 19.0 |
| food | 0.732 | medi | 0.464 | clth | 19.2 |
| educ | 0.792 | trans | 0.491 | recr | 19.5 |
| recr | 0.815 | hous | 0.568 | furn | 19.6 |
| furn | 0.892 | recr | 0.569 | trans | 19.7 |
| clth | 1.053 | educ | 0.678 | misc | 19.9 |


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    ${ }^{1}$ Atkinson and Stiglitz [1972], for example, computed optimal tax rates in a single consumer economy. their computation showed that the optimal tax rates diverged siginificantly from uniformity. Deaton [1977] pointed out the desirability of food subsidy for equity reason. Harris and MacKinnon [1979] found that food should be taxed very heavily in a one-consumer model

[^1]:    ${ }^{4}$ Normally, the value of $I$ is assumed to be zero, i.e. there is no lump sum income and transfer allowed in the optimal taxation problem.
    ${ }^{5}$ If we define the net consumption of leisure $x_{n}$ by

    $$
    x_{n} \equiv q_{n}-L,
    $$

    the budget constraint can be rewritten as

    $$
    \sum_{i=1}^{n-1} p_{i} q_{i}+p_{n} x_{n}=I .
    $$

    Then $-x_{n}$ is the labor supply and $-p_{n} x_{n}$ is the wage income of the consumer. Thus our utility maximization problem is identical to the familiar income leisure choice of text book microeconomics.

[^2]:    ${ }^{6}$ Since leisure is assumed to be nontaxable, we have $p_{n}=p_{n}^{0}$.
    ${ }^{7}$ See Judge et. al.[1985] Appendix B.
    ${ }^{8}$ This is due to the homogeneity property of the demand system. Notice that a tax on the endowment of leisure is a lump sum tax.

[^3]:    ${ }^{9}$ See Hatta [1991] for proofs and more details.
    ${ }^{10}$ Samuelson [1951]

[^4]:    ${ }^{11}$ A uniform taxation on all goods (excluding leisure) is analytically equivalent to a proportional wage taxation. This is easily seen from the fact that the consumer's budget and revenue constraints are undisturbed by shifting tax structure from a uniform commodity taxation to a wage taxation.
    ${ }^{12}$ One difference we have to note is the point of evaluation of the wage elasticities compared. With inverse wage elasticity rule, elasticities are evaluated at the optimal state, whereas with Corlett and Hague rule, they are evaluated at the initial uniform tax equilibrium. For threegood (two commodities and labor) economy, the difference of evaluation point does not change the result. However, for a general $n$-commoditiy economy, the elasticies could assume different orders when the evaluation point is altered. As a result, the elasticities evaluated at the initial uniform tax equilibrium could not be a perfect guide about the optimal tax structure even when the cross elasticities are all zero.

[^5]:    ${ }^{13}$ See Sadka [1977].
    ${ }^{14}$ Of course, the optimal taxation will attain higher welfare. However, it cost more to administer the optimal taxation since different tax rates apply to different commodities. If the deadweight loss of the uniform taxation is not too large compared to the one of the optimal taxation, the uniform taxation can be a better choice if the whole costs are considered.

[^6]:    ${ }^{15}$ The price index is defined to attain the minimum utility level with one unit of income.
    ${ }^{16}$ See Deaton and Muellbauer [1980], and Anderson and Blundel [1983, 1985].

[^7]:    ${ }^{17}$ See Hausman and Taylor [1981], and Kang [1987].
    ${ }^{18}$ The specification should be tested statistically. Here we simply assumed the mixed model.

[^8]:    The specification test is on the top of our agenda.

[^9]:    ${ }^{19}$ We assumed that the subsistent leisure is eight hours per day. We varied the number of hours only to get the similar result.
    ${ }^{20} \mathrm{By}$ the extended model, we mean the model that includes the income-leisure choice.

[^10]:    ${ }^{21}$ See Asano [1997] for more discussion and details of this point.
    ${ }^{22}$ See Goldman and Uzawa (1964, theorem 5).
    ${ }^{23}$ See Asano [1997] for more details.

[^11]:    ${ }^{24}$ We evaluated the substitution matrix at sample mean values of the log prices and total endowment, and derived standard errors by the $\delta$-method (Rao [1972]).
    ${ }^{25}$ In Borjas and Heckman [1979], the range was from 0.04 to 0.20 , in Killingsworth [1983] it was from 0.14 to 0.20 . More recently, Pencavel [1986] surveyed fourteen major empirical studies. He reported that five yielded the estimates with wrong sign, which is inconsistent with theory. Of the acceptable estimates, the largest was 0.84 and the smallest was 0.04 . Excluding the extreme values, the average was 0.11 .

[^12]:    ${ }^{26}$ For example see Barnett [1979], and Browining and Meghir [1991].
    ${ }^{27}$ Corlett and Hague [1953] could order two tax rates according to the values of the substitution elasticities in a three-good model. Their result does not extend to the n-good economy as we mentioned in Section 2.2.

[^13]:    ${ }^{28}$ See Varian [1984] for the definitions of CV and EV.
    ${ }^{29}$ We did not consider the equity issues in the present paper. However, we expect that the similar results hold if non linear income tax is used with commodity taxation.

