

# The Cyclicalities of Entry and Exit: A General Equilibrium Analysis with Imperfect Information

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## Abstract

The US establishment exit rate is acyclical. This poses a challenge to canonical models of industry dynamics-e.g., Hopenhayn (1992)-which imply a strongly counter-cyclical exit rate. To reconcile this gap between theory and data, imperfect information is introduced. Potential entrants have imperfect information about their productivity, leading to a signal extraction problem. When the volatility of idiosyncratic productivity dominates that of aggregate-as we observe in the micro data, potential entrants overestimate their productivity, and the value of entering, in booms. This amplified entry further increases factor prices and crowds out marginal incumbents, making the exit rate almost acyclical. The imperfect information mechanism proposed here also yields three testable implications: (i) entry is more cyclical in the industries where idiosyncratic components dominate; (ii) plant entry by new firms is more cyclical than that by existing firms; (iii) plants established by new firms during booms are more likely to exit rapidly. We show that all three predictions are consistent with the data.

**Keywords:** Entry and exit, Plant Dynamics, Business Cycles, Imperfect Information

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<sup>†</sup>[Link to the latest version.](#)

# 1. Introduction

It is well known that entry and exit play important roles in the cyclical behavior of aggregate job flows and productivity growth.<sup>1</sup> Recently, industry equilibrium model a la [Hopenhayn \(1992\)](#) has been extended to account for the dynamics of entry and exit over the business cycle.<sup>2</sup> Canonical models generate modestly pro-cyclical entry and counter-cyclical exit (e.g. [Clementi and Palazzo \(2015\)](#) and [Clementi et al. \(2015\)](#)). However, the establishment exit rate is acyclical and the entry rate is strongly procyclical in the data, according to Business Dynamics Statistics (1980–2012) and [Lee and Mukoyama \(2015a\)](#).

To reconcile this gap between the model and data, imperfect information is introduced. Potential entrants do not have a perfect information about its productivity. As a result, potential entrants face a signal extraction problem. When the volatility of idiosyncratic productivity dominates that of aggregate—as we observe in the micro data, potential entrants are likely to overestimate their idiosyncratic productivity in booms.<sup>3</sup> This amplified entry further increases factor prices and crowds out marginal incumbents. As a result, the exit rate becomes much less cyclical.

In addition to informational friction, our model distinguishes itself from the previous models in the heterogeneous firm literature by jointly incorporating (i) non-convex adjustment costs in capital to account for the plant level investment dynamics, (ii) non-convex adjustment costs in labor to account for the job reallocation rate observed in the data, (iii) life cycle property of plants, and (iv) realistic elasticity of labor supply.<sup>4</sup>

Moreover, our model sheds new insights on the recently popular learning models in macroeconomics. One of the shortcomings of the learning models in the literature is that once economic agents are allowed to observe the equilibrium aggregate price precisely, they quickly learn about the aggregate state, making the propagation from imperfect information disappear rapidly.<sup>5</sup> Whereas in our model, even though potential entrants are allowed to

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<sup>1</sup>For example, [Davis et al. \(1996\)](#) and [Foster et al. \(2001\)](#).

<sup>2</sup>Notably including [Lee and Mukoyama \(2015b\)](#), [Clementi and Palazzo \(2015\)](#), [Clementi et al. \(2015\)](#) and [Siemer \(2014\)](#).

<sup>3</sup>According to [Castro et al. \(2015\)](#) and [Cooper and Haltiwanger \(2006\)](#), in the U.S. manufacturing sector, idiosyncratic shock (to the profitability of plants) is around 5~6 times more volatile than aggregate shock.

<sup>4</sup>For example, [Clementi et al. \(2015\)](#) does not have a labor adjustment cost and uses an infinite Frisch elasticity. [Bloom et al. \(2014\)](#) does not have an entry and exit margin and also uses an infinite Frisch elasticity.

<sup>5</sup>For example, [Venkateswaran \(2014\)](#), [Li and Weinberg \(2003\)](#) and [Lorenzoni \(2009\)](#).

observe the equilibrium aggregate prices accurately, they still cannot perfectly learn about the aggregate state. This is due to the fact that individual plants' factor demand decision (e.g. employment and investment) is dynamic. By contrast, for example, if plants' labor demand decision was static, observing the spot labor market equilibrium wage allows the potential entrants to infer the true aggregate state easily.

The key parameters of the model are tightly disciplined by (i) the characteristics (e.g., relative productivity and employment size) of entering and exiting plants (ii) cross-sectional distribution of plant-level investment rates, and (iii) job flows rates in the manufacturing sector.

The model economy driven by aggregate TFP shocks generates a realistic behavior of entry and exits. The average entry rate is 7.6% during booms (which is 8.1% in the Annual Survey of Manufacturers). The average entry rate during recessions is 4.5% (3.4% in the data). The exit rate from the model is acyclical: the correlation coefficient with cyclical output is  $-0.09$  ( $-0.02$  in the data). To isolate the role of information frictions, we also simulate the model under the perfect information—potential entrants know their productivity precisely. The entry is much less cyclical and exit rate is strongly counter-cyclical under the perfect information. The average entry rate in booms is 6.3% (much lower than 8.1% in the data). That in recessions is 5.9% (higher than 3.4% in the data). The correlation between the exit rate and output under full information is  $-0.49$ .

The model with imperfect information also generates *age-dependent* cyclical job flows that is consistent with the data. According to [Foster et al. \(2006\)](#), the time series volatility of job flows—both creation and destruction—of young (age  $\leq 3$ ) plants is much larger than that of old plants. Moreover, the job creation rate is much more volatile than job destruction rate for young plants. Our model, by matching the volatility of the entry margin in the data, can also reproduce this pattern of age-dependent job flows. Many young plants whose productivity is not so high continue to operate because entry costs are sunk, are higher than the fixed costs of operating, and investment is partially irreversible. Not all the jobs created by young firms are destroyed immediately. As a result, the job destruction of young plants is less volatile than that of job creation. By contrast, according to the model with full information, young plants exhibits the volatility of job creation which is similar to that of destruction.

While it is difficult to directly observe the information frictions that firms face, the mechanism proposed in this paper provides at least three testable implications: (i) plant entry by new firms (who have less information about their productivity) is more cyclical than that by existing firms; (ii) plants established by new firms during booms are more likely to exit; (iii) entry is more cyclical in industries where idiosyncratic components dominate. We show that all three predictions are consistent with the data.

Specifically, (i) according to the Business Dynamics Statistics, the cyclical variation of establishment entry by new firms is 50% more volatile than that by existing firms; (ii) according to the Business Dynamics Statistics, the one-year exit rate of plants established by new firms during booms is 7% higher than that during recessions; (iii) across 2-digit manufacturing industries, the cross-sectional correlation coefficient between the cyclicality of entry and the volatility of idiosyncratic component of sales growth is 0.38 in the Annual Survey of Manufacturers.

This paper is closely related to various recent literature that extends the [Hopenhayn \(1992\)](#) industrial equilibrium model by adding business cycle fluctuations in competitive equilibrium. [Clementi and Palazzo \(2015\)](#), [Clementi et al. \(2015\)](#) and [Lee and Mukoyama \(2015b\)](#) have developed variants of this model to investigate the business cycle behavior of entry, exit, and plant dynamics. The first two of these papers are focused on the role of entry and exit as a business cycle propagation mechanism and investigate the role of the net entry margin in explaining the slow recovery following the Great Recession. [Clementi and Palazzo \(2015\)](#), by omitting goods market clearing, and [Clementi et al. \(2015\)](#), by using an infinite Frisch elasticity of labor supply, both have procyclical entry and countercyclical exit. [Lee and Mukoyama \(2015b\)](#), like this paper, try to explain procyclical entry and acyclical exit jointly. Their main mechanism is an exogenous cyclical entry cost while the main mechanism of our paper is instead the imperfect information of potential entrants. Another distinguishing feature of our paper is that it includes non-convex adjustment costs in both capital and labor and disciplines general equilibrium forces with reasonable numbers for risk aversion and the elasticity of labor supply.<sup>6</sup>

The information friction used in this paper is that some economic agents cannot dis-

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<sup>6</sup>[Clementi and Palazzo \(2015\)](#) has only capital adjustment costs. [Clementi et al. \(2015\)](#) has only capital adjustment costs and uses an infinite Frisch elasticity. [Lee and Mukoyama \(2015b\)](#) does not have capital and uses linear utility in consumption.

tinguish between aggregate and idiosyncratic productivity. This has been extensively used in various strands of business cycle research, especially in the context of monetary non-neutrality and persistent real effects from nominal shocks. In [Lucas \(1972\)](#), the inability of agents to distinguish between a pure monetary shock and an island-specific real shock generates monetary non-neutrality. [Mackowiak and Wiederholt \(2009\)](#) use a rational inattention argument to provide a mechanism by which a nominal shock has persistent real effects. [Hellwig and Venkateswaran \(2014\)](#), in a price setting context, investigate under which conditions will an agent’s inability to distinguish between aggregate and idiosyncratic shocks amplify a nominal shock.

There are also papers using a similar information friction as a propagation mechanism with respect to an aggregate productivity shock. [Venkateswaran \(2014\)](#) used a firm’s inability to distinguish between aggregate and idiosyncratic productivity as one way to resolve the [Shimer \(2005\)](#) puzzle in the context of a search and matching model. [Li and Weinberg \(2003\)](#) also use a similar information friction to explain the different cyclical sensitivities of investment across small and large firms. But our paper differs from these papers in two dimensions. One, potential entrants here can learn about the aggregate status by observing current equilibrium wages and the aggregate capital stock while in the aforementioned papers firms cannot observe any current period aggregate equilibrium variables. Two, in our paper, there is interaction between incumbents who have full information and potential entrants with imperfect information through the goods and labor market clearing conditions.

The paper is organized as follows. In [Section 2](#), we recap the cyclical properties of plant entry and exit in the manufacturing sector using data from the BDS. In [Section 3](#), the model economy is described formally. [Section 4](#) discusses the calibration and various resulting life cycle profiles of plants. [Section 5](#) covers the quantitative results, provides insight on the model mechanism through impulse response exercises, and examines the model’s implications for the cyclical behavior of entry and exit, the age dependent cyclical behavior of job flows, and the role of the net entry margin as an internal propagation mechanism. In [Section 6](#), we check some testable implications of our informational story by looking at plant entry and exit depending on firm age and the corresponding 2-digit manufacturing sector’s entry rate. [Section 7](#) provides additional robustness checks before concluding in [Section 8](#).

## 2. The Cyclical Behavior of Establishment Entry and Exit in the Manufacturing Sector

Lee and Mukoyama (2015a) have documented that in the U.S. manufacturing sector the establishment entry rate is procyclical and the exit rate is acyclical. Their findings rely on the Annual Survey of Manufacturers over the sample period 1972 to 1997.

Table 1: Manufacturing firm entry and exit rates

	Boom	Recession	Total avg.	<i>p</i> -value
Entry(birth)	8.1%	3.4%	6.2%	0.023
Exit(death)	5.8%	5.1%	5.5%	0.371

Note: This table is from Table 2 of Lee and Mukoyama (2015a). Entry (exit) rate is defined as the number of entering (exiting) plants as percentage of the total number of plants each period. *p*-values are from the *t*-test of the mean difference in entry and exit rate between boom and recession years.

Table 1 is taken from Lee and Mukoyama (2015a), which breaks down establishment entry and exit rates over different parts of the business cycle. Here, ‘boom’ years are those when the manufacturing sector’s output growth rate is higher than the sample average manufacturing output growth rate, while ‘recession’ years are those where manufacturing output growth is below the sample average. From Table 1 it is clear that the entry rate varies considerably through the business cycle but there is no significant variation in the exit rate between boom and recession years.

We checked the robustness of their findings using the establishment entry and exit rate derived from the BDS (Business Dynamics Statistics) data set. Compared to the ASM, which is biased towards large employment plants, the BDS includes all plants with a positive payroll. The BDS is also available for a larger sample period, 1977 to 2012. As entry and exit rates in a given year of the BDS measure what has occurred from the previous year’s March to the current year’s March, we need to construct a properly re-timed annual business cycle indicator for the manufacturing sector from high frequency data. This can be accomplished by using monthly industrial production of the manufacturing sector and quarterly aggregate

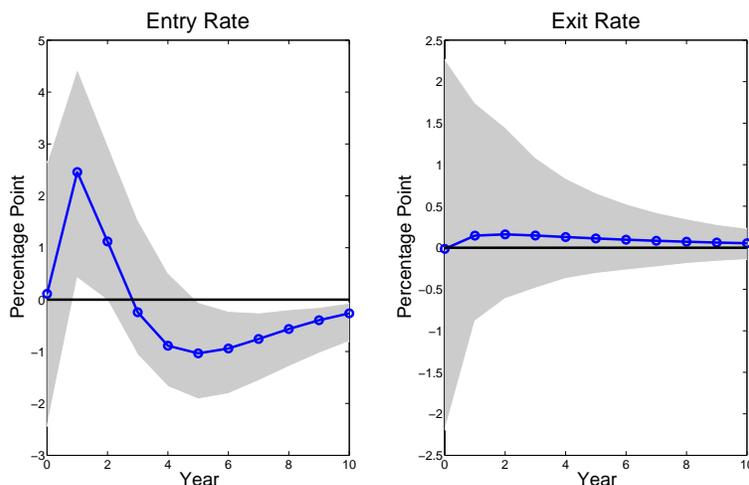
employment in the manufacturing sector.<sup>7 8</sup>

Table 2: The cyclical behavior of manufacturing entry and exit rates

	Industrial Production	Employment
$\text{corr}(\text{entry}_t, \text{indicator}_t)$	0.35**	0.49**
$\text{corr}(\text{exit}_t, \text{indicator}_t)$	-0.02	0.01
$\text{corr}(\text{exit}_t, \text{indicator}_{t-1})$	0.5***	0.17
$\text{corr}(\text{exit}_t, \text{indicator}_{t-2})$	0.46***	0.27

Note: *Indicator* represents either industrial production or employment. \*\* means the correlation is significant at 5% and \*\*\* means significant at 1%.

Figure 1: Response of the entry and exit rates to a productivity shock



Note: The shaded region represents the bootstrapped 90% confidence interval.

From Table 2 the resulting establishment entry rate is procyclical and the exit rate is acyclical. If we look at the correlation of the exit rate with the generated lagged cyclical

<sup>7</sup>For the output measure, we used the industrial production index for the manufacturing sector. From this monthly industrial production series, we constructed a re-timed annual measure that is consistent with March to March timing in the BDS. In case of the employment measure, we used BLS’s manufacturing sector employment index based on SIC classification. From the quarterly index, we again constructed a re-timed annual measure.

<sup>8</sup>When we use output as the cyclical indicator, the sample period is 1980 to 2012 and we excluded 2002 observations. That is because the BDS is mainly constructed based on the Census Bureau’s Business Register and starting from 2002 new identification numbers, especially for multi-unit plants, were implemented. This raises concerns about the possibility of spurious entry or exit in 2002 in the BDS. When we use employment as the cyclical indicator, the sample period is 1980 to 2001. This is because the BDS is based on the SIC classification system, and the BLS stopped providing SIC-based quarterly manufacturing sector employment after 2002.

indicator, there is still no evidence of countercyclical exit. Rather, in the case of industrial production, the establishment exit rate is significantly procyclical with respect to the lagged indicator.

It is also informative to look at the dynamic response of establishment entry and exit rates to a productivity shock. To this end, we construct two VAR systems. Each is composed using re-timed real output per hour in the manufacturing sector and either the establishment entry or exit rate.<sup>9</sup> The identification scheme for the productivity shock in both cases is that productivity can have a contemporaneous effect on either the entry or exit rate, but a shock to the entry or exit rate cannot affect productivity immediately - these shocks can only affect productivity from next period onwards. Figure 1 shows the response of entry and exit rates with respect to a one standard deviation positive shock in productivity within their respective VAR.

This impulse response exercise reassures us that the establishment entry rate is procyclical and the exit rate is acyclical. The entry rate shows especially interesting dynamics. It overshoots during the early stage of the boom and then falls sharply before recovering. In the quantitative exercise we will test whether the model economy can generate this pattern in entry dynamics through incorporating a learning problem among potential entrants.

### 3. Model

The model economy is composed of incumbent plants, a fixed measure of potential entrants and a representative household. Incumbent plants produce output using capital and labor in the presence of both capital and labor adjustment costs. Potential entrants observe wages and aggregate capital, plus an exogenous signal which contains information about both aggregate economic conditions and their idiosyncratic status, and then solve a Kalman filter problem to attempt to disentangle aggregate and idiosyncratic productivity. Based on their resulting estimates, they make an optimal entry decision. The representative household owns all the plants in the economy and makes consumption and labor supply decisions.

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<sup>9</sup>Because of the limited sample size, it is hard to get stable estimates from VARs with 3 endogenous variables (real output per hour, entry rate, exit rate). Hence, we estimate two VARs with 2 endogenous variables separately.

### 3.1. Incumbents

Time is discrete. At time  $t$ , any price-taking incumbent plant  $i$  produces a homogeneous good using a decreasing returns to scale production function  $y_{t,i} = \exp(z_t + x_{t,i})(k_{t,i}^\alpha n_{t,i}^{1-\alpha})^\theta$ .  $z_t$  and  $x_{t,i}$  are aggregate and idiosyncratic productivity, respectively. The stochastic processes for these two shocks are given as follows.

$$z_{t+1} = \rho_z z_t + \epsilon_{z,t+1} \quad (1)$$

$$x_{t+1} = \rho_x x_t + \epsilon_{x,t+1} \quad (2)$$

Here,  $\epsilon_{z,t+1} \sim N(0, \sigma_z^2)$  and  $\epsilon_{x,t+1} \sim N(0, \sigma_x^2)$ . Denote the conditional distribution of  $x_{t+1}$  given  $x_t$  as  $H(x_{t+1}|x_t)$ . In contrast to potential entrants, incumbent plants can observe each of the  $z_t$  and  $x_{t,i}$  separately. This means that incumbent plants can accurately form expectations with respect to future equilibrium prices, the evolution of the distribution of plants, and therefore their own expected profit from continuing operation.

At the beginning of the each period, an individual incumbent plant is characterized by its predetermined level of capital( $k$ ), labor( $n_{-1}$ ), and its current idiosyncratic productivity level( $x$ ). At the beginning of the period, the distribution of incumbent plants over  $(n_{-1}, k, x)$  constitutes one aggregate state variable along with the aggregate productivity level  $z$  and the distribution of potential entrants over their prior beliefs.

Given these state variables, incumbent plants also observe their stochastic fixed operating cost,  $\xi$ , which is drawn from a time-invariant distribution and is i.i.d. across both time and plants. If the expected profit from continuing to operate is large enough to justify paying this output-denominated fixed operating cost then plants will do so and remain as incumbents at the beginning of the next period. Otherwise, they will not pay the fixed operating cost and permanently shut down from next period on. Note that shut down occurs at the end of the period, so current incumbents produce this period regardless, but due to the labor adjustment costs their employment decisions depend on whether they intend to shut down or not.

When the incumbent chooses to continue to operate, it has to choose this period's labor and investment. In order to discipline plants' investment and labor adjustment decisions to be consistent with micro level factor adjustment behavior, we will introduce partial irre-

versibility in capital adjustment and linear costs in labor adjustment.

$$k' = k(1 - \delta) + i \quad (3)$$

$$AC^k = (1 - p_s)|i|\mathbb{1}_{(i < 0)} \quad (4)$$

$$AC^n = |n - n_{-1}|c_n \quad (5)$$

Specifically, capital depreciates at rate  $\delta$ , the resale price of installed capital is discounted by fraction  $1 - p_s$ , and whenever plants adjust labor they pay  $c_n$  per unit of adjustment. Unlike the capital adjustment, labor adjustment is immediately reflected in current period production. The sluggish and forward-looking factor adjustment behavior introduced by these adjustment costs will detach equilibrium wage dynamics from the dynamics of aggregate productivity. Consequently potential entrants, even after observing the equilibrium wage, cannot fully infer the current aggregate status, a point which we will return to.

If a plant decides to not pay the fixed operating cost, it first determines current period labor demand taking into account that after the current period's production the plant will fire all the labor hired and pay the associated labor adjustment cost. After production, these plants that do not pay their operating cost exit the market with the resale value of the remaining capital stock after depreciation.

Denote the incumbent distribution over  $(n_{-1}, k, x)$  as  $\Gamma$ , the distribution of potential entrants over  $(a, \mu_{t|t-1}^z, \mu_{t|t-1}^q)$ <sup>10</sup> as  $\Omega$  and define  $\Lambda = (\Gamma, \Omega)$ . Now we can summarize the incumbent's optimization problem using the following value functions.

$$V(n_{-1}, k, x; z, \Lambda) = \int \max\{V_c(n_{-1}, k, x; z, \Lambda) - \xi, V_x(n_{-1}, k, x; z, \Lambda)\}dG(\xi) \quad (6)$$

$$V_c(n_{-1}, k, x; z, \Lambda) = \max_{\{i, n\}} y - w(z, \Lambda)n - i - AC^k(k, i) - AC^m(n_{-1}, n) + \mathbb{E}[d(z', \Lambda')V(n, k', x'; z', \Lambda')] \quad (7)$$

$$V_x(n_{-1}, k, x; z, \Lambda) = \max_{\{n\}} y - w(z, \Lambda)n - AC^m(n_{-1}, n) - AC^m(n, 0) + p_s(1 - \delta)k \quad (8)$$

$$\chi(n_{-1}, k, x; z, \Lambda) = G(\xi \leq \max\{V_c - V_x, 0\}) \quad (9)$$

$d(z', \Lambda')$  represents the state contingent discount factor. Both the spot labor market equilibrium wage and the state contingent discount factor are determined jointly with the representative household's optimization problem and the market clearing conditions.  $\chi(n_{-1}, k, x; z, \Lambda)$

<sup>10</sup>Where  $a$  is sum of the their potential idiosyncratic productivity and current period aggregate productivity.  $\mu_{t|t-1}^z, \mu_{t|t-1}^q$  are prior beliefs about aggregate productivity and potential idiosyncratic productivity respectively. Details will be provided in section 3.2.

represents the probability that plants whose beginning of period state variables are  $(n_{-1}, k, x)$  continue operating. Because the operating cost is given stochastically, the optimal exit decision is represented in terms of probability.

### 3.2. Potential Entrants

There exists a fixed measure  $M$  of potential entrants in every period. Each period, based on their current information set, potential entrants form expectations about profits from operating next period onwards. For a potential entrant to accurately evaluate their expected profit from entry they need both their idiosyncratic productivity  $(x_{t+1,i})$  and aggregate productivity  $(z_{t+1})$  next period, the first period they would be able to produce. As these are impossible to observe today, this implies that the expectations formed by potential entrants today and the information those expectations are taken with respect to will be a key part of their economic decision-making.

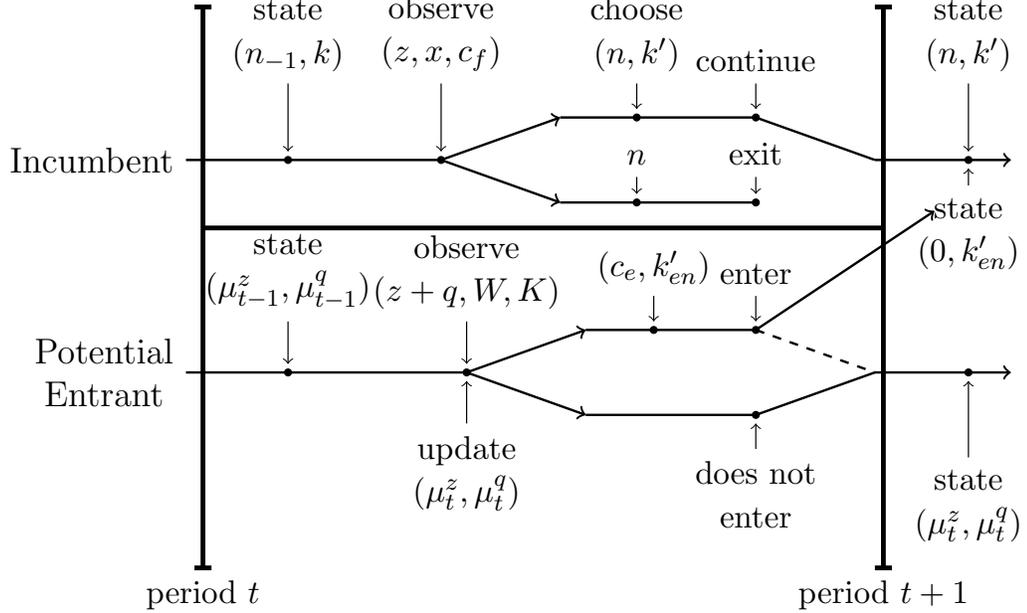
One critical question is whether potential entrants have the same amount of information as incumbent plants. The standard approach taken so far in this literature<sup>11</sup> is that potential entrants get a signal about their own idiosyncratic productivity and they can observe current aggregate productivity, just as incumbent plants do. That is, a potential entrant is modeled as having the same information set as incumbent plants. This paper departs from this symmetric information structure between potential entrants and incumbents. Specifically, potential entrants here only receive a signal on the sum of their own potential idiosyncratic productivity and current aggregate productivity.

Figure 2 diagrams how the information set of potential entrants influences their decisions compared to incumbent firms. Potential entrants start the period with prior beliefs over aggregate productivity,  $\mu_{t-1}^z$ , and their own idiosyncratic productivity,  $\mu_{t-1}^q$ , based upon the history of signals they've observed. As mentioned in the previous section, there exists a non-degenerate distribution of potential entrants over these state variables. Incumbents then observe a different information set than do potential entrants. Incumbents use their information to adjust their employment and capital stock and decide whether to shut down or not, while potential entrants use their information to update their beliefs and decide

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<sup>11</sup>Mostly extensions of [Hopenhayn \(1992\)](#) with business cycle fluctuations, including [Lee and Mukoyama \(2015a\)](#), [Clementi and Palazzo \(2015\)](#), and [Clementi et al. \(2015\)](#).

Figure 2: Timeline of the model economy



whether or not to enter. If they do enter, they pay the output denominated entry cost  $c_e$  and buy the fixed amount of capital required to enter,  $k'_{en}$ .

The stochastic processes for a potential entrants' potential idiosyncratic productivity ( $q_t$ ), and how it evolves into an actual idiosyncratic productivity next period in the case of entry, are given by the following AR(1) specifications.

$$q_{t+1} = (1 - \rho_q)\bar{q} + \rho_q q_t + \epsilon_{q,t+1} \quad (10)$$

$$x_{t+1} = \rho_x q_t + \epsilon_{x,t+1} \quad (11)$$

Here,  $\bar{q} < 0$  is the long-run mean of the process and  $\epsilon_{q,t+1} \sim N(0, \sigma_q^2)$ ,  $\epsilon_{x,t+1} \sim N(0, \sigma_x^2)$  are the realized innovations.

Denote the conditional distribution of  $x_{t+1}$  given  $q_t$  as  $H(x_{t+1}|q_t)$  and the conditional distribution of  $q_{t+1}$  given  $q_t$  as  $J(q_{t+1}|q_t)$ . The stationary distribution of the potential entrants in terms of their would-be productivity is stochastically dominated by the stationary distribution of the incumbent's idiosyncratic productivity process. Combined with the mean-reverting property of  $x$  - incumbent idiosyncratic productivity - this generates an increasing age profile in terms of plant productivity. To keep the distribution of potential entrants stationary, if one decides to enter it is replaced by a new potential entrant who inherits the same potential productivity  $q$ . In terms of the distribution of  $q$  this is identical to an econ-

omy where a fixed measure of potential entrants draw their idiosyncratic productivity from the long-run stationary distribution implied by (10). Note that a potential entrant  $i$  cannot observe their current period  $q_{t,i}$  but they can only observe the sum of aggregate productivity and  $q_{t,i}$ . We denote that combined signal as  $a_{t,i}$ .

$$a_{t,i} = q_{t,i} + z_t \tag{12}$$

While potential entrants cannot observe the components directly, they are aware of the structure (long-run mean, persistence, and variance) of each of the component processes.

For example, in a given period if there is positive shock to aggregate productivity then every potential entrant will receive a more positive signal but they cannot distinguish whether it resulted from aggregate productivity or potential idiosyncratic productivity. If they could observe the aggregate shock directly, they could account for its effect on future wages and consumption<sup>12</sup> when they calculate expected profit from entry. But given that a potential entrant cannot observe aggregate productivity separately from their signal, they attribute some portion of the positive movement in the signal to the idiosyncratic shock and underestimate the movements in future equilibrium wages and consumption. This leads potential entrants to overestimate their expected profit from entry when the economy booms. Conversely, potential entrants underestimate their expected profit from entry when the economy slumps.

Fluctuations in aggregate productivity generate fluctuations in the equilibrium wage but changes in an individual potential entrant's potential profitability would not have any effect on aggregate variables. Therefore potential entrants can get some information about aggregate economic conditions by observing the current period wage. Compared to previous papers that used similar information frictions, this paper differs in that even after observing equilibrium period wages from the aggregate spot market there still remains a problem of imperfect information.

Potential entrants can also learn about the aggregate state by observing the beginning of period aggregate capital stock. This economy is characterized by a non-degenerate distribution of plants and the aggregate fluctuation. To forecast equilibrium price dynamics

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<sup>12</sup>Given that the representative household is risk averse, the value of the final good across different aggregate states is determined by the goods market clearing consumption level in each aggregate state.

consistently, agents in this economy rely on the law of motion for the distribution of plants. Under a bounded rationality assumption agents approximate the distribution with a bounded set of its moments. In this economy, agents use the mean of the capital distribution - they are always aware of the current period aggregate capital stock. Given that yesterday's capital choice was a function of yesterday's aggregate productivity and productivity is persistent, capital today gives some information about economic conditions. So it is natural that potential entrants use current period capital stock in price forecasting but also in estimating the current period's aggregate productivity. Potential entrants learn about aggregate productivity by using the observed wage and aggregate capital stock to follow these projection equations.<sup>13</sup>

$$\log w = \beta_{w,c} + \beta_{w,z}z + \epsilon_w, \text{Var}(\epsilon_w) = \sigma_w^2 \quad (13)$$

$$\log K = \beta_{K,c} + \beta_{K,z}z + \epsilon_K, \text{Var}(\epsilon_K) = \sigma_K^2 \quad (14)$$

After some potential entrant  $i$  observes  $(a_{t,i}, w_t, K_t)$ , they try to estimate  $(z_t, q_{t,i})$  which are necessary to precisely estimate the expected profit from entering the market. Potential entrants solve a Kalman filter problem comprised of the following measurement and transition equation.

$$\underbrace{\begin{bmatrix} a_t \\ \log w_t \\ \log K_t \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} 1 & 1 \\ \beta_{w,z} & 0 \\ \beta_{K,z} & 0 \end{bmatrix}}_{\beta} \begin{bmatrix} z_t \\ q_t \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \beta_{w,c} \\ \beta_{K,c} \end{bmatrix}}_{\beta_c} + \begin{bmatrix} 0 \\ \epsilon_w \\ \epsilon_K \end{bmatrix} \quad \text{Measurement Equation} \quad (15)$$

$$\begin{bmatrix} z_t \\ q_t \end{bmatrix} = \underbrace{\begin{bmatrix} \rho_z & 0 \\ 0 & \rho_q \end{bmatrix}}_T \begin{bmatrix} z_{t-1} \\ q_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ (1 - \rho_q)\bar{q} \end{bmatrix}}_{T_c} + \begin{bmatrix} \epsilon_z \\ \epsilon_q \end{bmatrix} \quad \text{Transition Equation} \quad (16)$$

$$\text{where } \begin{pmatrix} \epsilon_z \\ \epsilon_q \end{pmatrix} \sim i.i.d. \text{ N} \left[ 0, \underbrace{\begin{pmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_q^2 \end{pmatrix}}_{Q_t} \right]$$

One issue is whether the residuals in (13) and (14) follow a normal distribution or not.<sup>14</sup> According to the Jarque-Bera test with a null hypothesis that residuals from (13) or (14)

<sup>13</sup>Coefficients and the variance of residuals in these projection equations are determined as per a typical [Krusell and Smith \(1998\)](#) algorithm for price forecasting. Details are in the appendix.

<sup>14</sup>If either of the residuals does not follow a normal distribution then a Kalman filter is not the best filter but the best linear filter. In that case, if a potential entrant used a non-linear filter, for example a particle filter, they would get more efficient estimates.

follow a normal distribution using model generated time series, the null *cannot* be rejected at a 10% significance level for both (13) and (14).

Denote the time  $t$  information set *after* observing time  $t$  variables  $(a_t, w_t, K_t)$  using subscript  $t|t$ . Similarly the subscript  $t|t-1$  indicates the time  $t$  information set *before* observing time  $t$  variables  $(a_t, w_t, K_t)$ .  $\mu = [\mu^z \mu^q]'$  denotes a potential entrant's estimates for aggregate and idiosyncratic productivity given their information set. Then for each period these estimates and the potential entrant's perceived conditional distribution over next period's aggregate and idiosyncratic productivity are updated as follows.

$$\mu_{t|t-1}^z = \rho_z \mu_{t-1|t-1}^z \quad (17)$$

$$\mu_{t|t-1}^q = \rho_q \mu_{t-1|t-1}^q + (1 - \rho_q) \bar{q} \quad (18)$$

$$\mu_{t|t}^z = \mu_{t|t-1}^z + G_z (Y_t - \beta \mu_{t|t-1} - \beta_c) \quad (19)$$

$$\mu_{t|t}^q = \mu_{t|t-1}^q + G_q (Y_t - \beta \mu_{t|t-1} - \beta_c) \quad (20)$$

$$F(z'|a, \mu_{t|t-1}^z, \mu_{t|t-1}^q; w, K) = \mathbf{N}(\rho_z \mu_{t|t}^z, V_{t+1|t}^z) \quad (21)$$

$$F(x'|a, \mu_{t|t-1}^z, \mu_{t|t-1}^q; w, K) = \mathbf{N}(\rho_x \mu_{t|t}^q, V_{t+1|t}^x) \quad (22)$$

Here,  $G$  is the stationary Kalman gain and  $V_{t+1|t}$  is the stationary prediction variance. Derivations of  $G$  and  $V_{t+1|t}$  are provided in the appendix. Then, a potential entrant's expected value from entry and optimal entry decision are:<sup>15</sup>

$$V_{en}(a, \mu_{t|t-1}^z, \mu_{t|t-1}^q; w, K) = -k'_{en} + \int_{z'} \int_{x'} [d(z', \Lambda') V(0, k'_{en}, x'; z', \Lambda')] dF(x') dF(z') \quad (23)$$

$$\varepsilon(a, \mu_{t|t-1}^z, \mu_{t|t-1}^q; w, K) = \mathbb{1}_{\{V_{en}(a, \mu_{t|t-1}^z, \mu_{t|t-1}^q; w, K) \geq c_e\}} \quad (24)$$

where  $k_{en}$  is the fixed amount of capital that potential entrants must install if they decide to enter the market. If the expected value from entry  $V_{en}(a, \mu_{t|t-1}^z, \mu_{t|t-1}^q; w, K)$  is larger than the fixed entry cost  $c_e$ , then the potential entrant chooses to enter. If so they pay  $c_e$ , invest  $k_{en}$ , and next period become an age-0 incumbent plant with beginning of period labor and capital of 0 and  $k_{en}$  respectively. As each individual potential entrant has a different history for the composite signal  $\{a_{t,i}\}_{i=0}^{\infty}$  there is non-degenerate distribution of potential entrants over  $(a, \mu_{t|t-1}^z, \mu_{t|t-1}^q)$ . Denote it as  $\Omega$ .

The current period posteriors  $(\mu_{t|t}^z, \mu_{t|t}^q)$  of entrants are inherited as the priors of new potential entrants next period. Note that if the priors of new potential entrants were given

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<sup>15</sup> $F(x')$  and  $F(z')$  are abbreviations of  $F(x'|Y_t, \mu_{t|t-1}^z, \mu_{t|t-1}^q)$  and  $F(z'|Y_t, \mu_{t|t-1}^z, \mu_{t|t-1}^q)$  respectively.

by the unconditional mean of  $z$  and  $q$  then potential entrants' learning about the aggregate status would be further dampened and the quantitative effect of learning would be amplified. In that sense our current choice on the information set of new potential entrants is a conservative one.

### 3.3. Household

The representative household consumes the final good, makes a labor supply decision and owns all plants, including potential entrants.

$$\max_{\{c_t, n_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma_c} - 1}{1 - \sigma_c} - \gamma \frac{n_t^{1+1/\sigma_n}}{1 + 1/\sigma_n} \right] \quad (25)$$

$$\text{s.t. } c_t = w_t n_t + \Pi_t \quad (26)$$

$\Pi_t$  in (25) includes entry costs paid by entrants. Compared to previous general equilibrium models with explicit heterogeneous production units, one critical distinction in this paper is that the Frisch elasticity is finite. <sup>16</sup>

### 3.4. Recursive Competitive Equilibrium

A recursive competitive equilibrium consists of (i) value functions  $V$ ,  $V_c$ ,  $V_x$  and  $V_{en}$ , (ii) policy functions  $\chi$ ,  $n^{con}$ ,  $n^{ex}$ ,  $k'$ ,  $\mu'^z$ ,  $\mu'^q$ ,  $\varepsilon$ ,  $C$  and  $N$ , (iii) a wage  $w$  and state contingent discount factors  $d(z', \Lambda')$ ,  $\forall (z', \Lambda')$  and (iv) measures for incumbents,  $\Gamma$ , and potential entrants,  $\Omega$ , such that:

1.  $(V, V_c, V_x)$  solve (6)–(8), and  $(\chi, n^{con}, k', n^{ex})$  are the resulting policy functions for incumbent plants.
2.  $V_{en}$  is given as (23), and  $\varepsilon$  is the policy function for the optimal entry decision.  $(\mu'^z, \mu'^q)$  are given from (17)–(20).
3.  $(C, N)$  are the policy functions associated with the household optimization problem. (25)–(26).

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<sup>16</sup>For example, Khan and Thomas (2008) and Clementi et al. (2015) used an infinite Frisch elasticity. Our baseline model uses a Frisch elasticity of 1.5.

4. Labor market clears:

$$N(z, \Lambda) = \int n^{con}(n_{-1}, k, x; z, \Lambda) \chi(n_{-1}, k, x; z, \Lambda) d\Gamma(n_{-1}, k, x) \quad (27)$$

$$+ \int n^{ex}(n_{-1}, k, x; z, \Lambda) (1 - \chi(n_{-1}, k, x; z, \Lambda)) d\Gamma(n_{-1}, k, x)$$

5. Goods market clears:

$$C(z, \Lambda) = \int f(z, x, k, n^{con}(n_{-1}, k, x; z, \Lambda)) \chi(n_{-1}, k, x; z, \Lambda) d\Gamma(n_{-1}, k, x) \quad (28)$$

$$+ \int f(z, x, k, n^{ex}(n_{-1}, k, x; z, \Lambda)) (1 - \chi(n_{-1}, k, x; z, \Lambda)) d\Gamma(n_{-1}, k, x)$$

$$- \int (k'(n_{-1}, k, x; z, \Lambda) - k(1 - \delta)) \chi(n_{-1}, k, x; z, \Lambda) d\Gamma(n_{-1}, k, x)$$

$$+ \int p_s k(1 - \delta) (1 - \chi(n_{-1}, k, x; z, \Lambda)) d\Gamma(n_{-1}, k, x)$$

$$- \int \xi \chi(n_{-1}, k, x; z, \Lambda) d\Gamma(n_{-1}, k, x) dG(\xi)$$

$$- \int AC^n(n_{-1}, k, x; z, \Lambda) d\Gamma(n_{-1}, k, x)$$

$$- \int AC^k(n_{-1}, k, x; z, \Lambda) d\Gamma(n_{-1}, k, x)$$

$$- \mathbb{M} \left( \int (c_e + k_e) \varepsilon(a, \mu^z, \mu^x; z, \Lambda) d\Omega(a, \mu^z, \mu^x) \right)$$

6. Intra-temporal Euler equation holds:

$$\gamma N(z, \Lambda)^{1/\sigma_n} = w(z, \Lambda) C(z, \Lambda)^{-\sigma_c} \quad (29)$$

7. State contingent discount factors coincide with the household's marginal rate of substitution across aggregate states:

$$d(z', \Lambda') = \beta \frac{C'(z', \Lambda')^{-\sigma_c}}{C(z, \Lambda)^{-\sigma_c}}, \quad \forall (z', \Lambda') \quad (30)$$

8. The laws of motion for the distribution of incumbents and potential entrants are consistent:

(a) If  $(0 \notin \mathcal{N} \text{ or } k_{en} \notin \mathcal{K})$

$$\Gamma'(\mathcal{N}, \mathcal{K}, x') = \int_{\mathcal{B}(\mathcal{N}, \mathcal{K}; z, \Lambda)} \chi(n_{-1}, k, x; z, \Lambda) H(x'|x) d\Gamma(n_{-1}, k, x) \quad (31)$$

$$\mathcal{B}(\mathcal{N}, \mathcal{K}; z, \Lambda) = \{(n_{-1}, k, x) | n^{con}(n_{-1}, k, x; z, \Lambda) \in \mathcal{N} \text{ and } k'(n_{-1}, k, x; z, \Lambda) \in \mathcal{K}\}$$

(b) If  $(0 \in \mathcal{N}$  and  $k_{en} \in \mathcal{K})$

$$\begin{aligned} \Gamma'(\mathcal{N}, \mathcal{K}, x') &= \int_{\mathcal{B}(\mathcal{N}, \mathcal{K}; z, \Lambda)} \chi(n_{-1}, k, x; z, \Lambda) H(x'|x) d\Gamma(n_{-1}, k, x) \\ &\quad + \mathbb{M} \int \varepsilon(z + q, \mu^z, \mu^q; z, \Lambda) H(x'|q) d\Omega(z + q, \mu^z, \mu^q) \end{aligned} \quad (32)$$

(c)

$$\begin{aligned} \Omega'(z' + q', \mathcal{Z}, \mathcal{Q}) &= \int_{\mathcal{B}(\mathcal{Z}, \mathcal{Q}; z, \Lambda)} J(q'|q) d\Omega(z + q, \mu^z, \mu^q) \\ \mathcal{B}(\mathcal{Z}, \mathcal{Q}; z, \Lambda) &= \{(a, \mu^z, \mu^q) | \mu'^z(a, \mu^z, \mu^x; z, \Lambda) \in \mathcal{Z} \text{ and } \mu'^q(a, \mu^z, \mu^x; z, \Lambda) \in \mathcal{Q}\} \end{aligned} \quad (33)$$

### 3.5. How Does the Model Work?

Before presenting quantitative results, we briefly try to give some additional intuition for the mechanism of the model economy. In particular, we discuss how the imperfect information held by potential entrants amplifies entry with respect to aggregate productivity shocks and how this pattern of entry affects the exit margin through general equilibrium effects. Another important issue is how much information about the aggregate state potential entrants can infer through observing current period equilibrium wages from the spot labor market. If equilibrium wage dynamics closely track the dynamics of aggregate productivity then the informational friction that prevents potential entrants being able to distinguish between aggregate productivity and potential idiosyncratic productivity will be irrelevant. Therefore we also discuss why equilibrium wage dynamics are detached from aggregate productivity dynamics so that the signal problem faced by potential entrants remains non-degenerate.

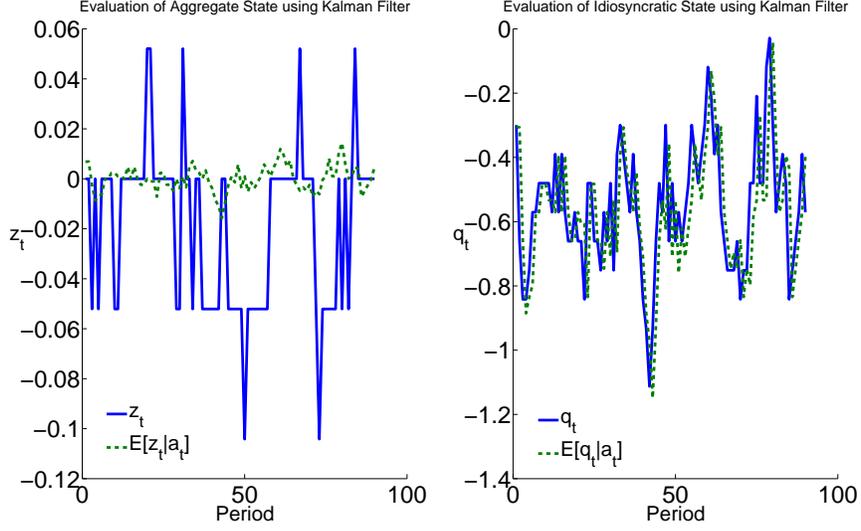
First consider what will happen if potential entrants can acquire information only from the exogenous signal<sup>17</sup>. Think of the following simple simulation exercise. We can generate artificial time series of  $\{z_t, q_t\}_{t=0}^{100}$  using the stochastic processes from our baseline model. Then by solving the potential entrant's Kalman filter problem, we can get estimates of  $z_t$  and  $q_t$  after observing  $a_t$  each period. In Figure 3, we plot the true variations in  $\{z_t, q_t\}$  together with variations in  $\{E[z_t|a_t]\}$  and  $\{E[q_t|a_t]\}$ .

What we can observe from this exercise is that potential entrants will misattribute most of the fluctuations in  $a_t$  as being driven by the idiosyncratic shock. While potential entrants cannot observe  $z_t$  and  $q_t$  separately, they know that innovations to  $q_t$  have a higher variance

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<sup>17</sup>See (12).

Figure 3: Stochastic simulation of potential entrant beliefs



Note: In each figure, the solid line represents the true variations in  $z_t$  and  $q_t$ . The dashed line represents variations in estimates of  $z_t$  and  $q_t$  using a Kalman filter after observing  $z_t + q_t$ .

than innovations to  $z_t$ . It is therefore natural to attribute the observed fluctuations in  $z_t + q_t$  as being caused by the more volatile component.

For example, return to the situation when the economy is hit by a positive aggregate productivity shock. Potential entrants partially misinterpret this as an increase in their potential idiosyncratic productivity. That is, after observing this period's  $z_t + q_t$ , by solving their Kalman filter problem potential entrants have  $\mu_{t|t}^z < z_t$ ,  $\mu_{t|t}^q > q_t$  and  $\mu_{t|t}^z + \mu_{t|t}^q = z_t + q_t$ . To get a better sense of how this underestimation of the aggregate productivity shock distorts potential entrants' expected profit from entry, consider the discounted value of operating profit in period  $t + 1$  from the perspective of a period  $t$  potential entrant. For simplicity, ignoring factor adjustment costs, it will be:

$$\beta \int_{z'} \int_{x'} [d(z', \Lambda') (\exp(z' + x') (k_{en}^\alpha n^{1-\alpha})^\theta - nw(z', \Lambda'))] dF(x' | \mu_{t|t}^q) dG(z' | \mu_{t|t}^z) \quad (34)$$

And recall that the stochastic processes for  $z_t$  and the transition from  $q_t$  to  $x_{t+1}$  are given as persistent AR(1)s.

$$z_{t+1} = \rho_z z_t + \epsilon_{z,t+1}$$

$$x_{t+1} = \rho_x q_t + \epsilon_{x,t+1}$$

Therefore, if the aggregate and idiosyncratic shocks are not equally persistent then  $\mu_{t|t}^z <$

$z_t, \mu_{t|t}^q > q_t$  implies that  $\mathbb{E}[z_{t+1} + x_{t+1} | \mu_{t|t}^z, \mu_{t|t}^q]$  is also biased. For example if  $\rho_x > \rho_z$  then  $\mathbb{E}[z_{t+1} + x_{t+1} | \mu_{t|t}^z, \mu_{t|t}^q] > \mathbb{E}[z_{t+1} + x_{t+1} | z, q]$  when the economy is hit by a positive shock and vice versa if the economy is hit by a negative shock. So if the idiosyncratic shock is more persistent than the aggregate shock, the entry response will be mechanically amplified. However, we make a conservative choice here so the model reflects the effect of the informational friction and not this mechanical amplification.<sup>18</sup> Specifically, we give the aggregate and idiosyncratic processes identical persistence. Because we have chosen  $\rho_z = \rho_x$ , even though potential entrants underestimate fluctuations in aggregate productivity,  $\mathbb{E}[z_{t+1} + x_{t+1} | \mu_{t|t}^z, \mu_{t|t}^q] = \mathbb{E}[z_{t+1} + x_{t+1} | z, q]$  still holds.

Note that the time  $t$  present value of period  $t + 1$  operating profit also depends on  $d(z_{t+1}, \Lambda_{t+1})$  and  $w_{t+1}(z_{t+1}, \Lambda_{t+1})$ . The value of the final good and equilibrium wages depend on the aggregate state (aggregate productivity and incumbent and potential entrants' distribution over individual state variables). Then any misunderstanding of the current period's aggregate productivity by potential entrants has an effect on their evaluation of next period's aggregate state in two ways. One, given the persistence of aggregate productivity, if potential entrants underestimate current aggregate productivity then they will also underestimate next period's aggregate productivity. Two, recall the law of motion for distribution of plants is:

$$\Lambda_{t+1} = \Phi(\Lambda_t, z_t)$$

where  $\Phi$  is an abbreviation of the equilibrium law of motion for the distribution of plants over their state variables (given as (31) ~ (33)). Potential entrants know  $\Phi$  and precisely observe  $\Lambda_t$ , but when potential entrants evaluate  $\Lambda_{t+1}$  they use  $\mu_{t|t}^z$  instead of the true  $z_t$  and  $\Phi(\Lambda_t, z_t) \neq \Phi(\Lambda_t, \mu_{t|t}^z)$ . Roughly speaking, potential entrants by underestimating variations in aggregate productivity also underestimate variations in aggregate state distributions.

It will be more illustrative if we state this in terms of the mean of the aggregate capital stock ( $K_t$ ) which was used as a proxy for  $\Lambda_t$  in our actual computation. When the economy is hit by a positive aggregate productivity shock, after solving their Kalman filter problem, potential entrants get  $z_t > \mu_{t|t}^z$  and as a result  $\Phi(K_t, z_t) > \Phi(K_t, \mu_{t|t}^z)$ . Denote the  $K_{t+1}$

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<sup>18</sup>We choose the persistence of aggregate productivity so the persistence of model generated output matches the persistence of actual manufacturing output. This calibration implies a less persistent aggregate productivity process than is typical in the literature. In that sense, our choice of persistence is conservative. Details regarding calibration are given in Section 4.

perceived by potential entrants as  $K_{t+1}^{en}$  and rewrite the potential entrants' perceived expected profit, (34), by substituting  $\Lambda$  with the aggregate capital stock  $K$ . To highlight the role of the informational friction, compare expected operating profit at period  $t+1$  under imperfect information, (35), with that under perfect information, (36).<sup>19</sup>

$$\beta \int_{z'} \int_{x'} [d(z', K'^{en}) (\exp(z' + x')(k_{en}^\alpha n^{1-\alpha})^\theta - nw(z', K'^{en}))] dF(x'|\mu_{t|t}^q) dG(z'|\mu_{t|t}^z) \quad (35)$$

$$\beta \int_{z'} \int_{x'} [d(z', K') (\exp(z' + x')(k_{en}^\alpha n^{1-\alpha})^\theta - nw(z', K'))] dF(x'|q) dG(z'|z) \quad (36)$$

Given the persistence of aggregate productivity,  $G(z'|z)$ <sup>20</sup> is dominated in a first order stochastic sense by  $G(z'|\mu_{t|t}^z)$ . According to our stochastic simulation, equilibrium consumption (recall that in equilibrium,  $d(z', K') = \beta \frac{C'(z', K')^{-\sigma_c}}{C^{-\sigma_c}}$ ) and wages are increasing in both aggregate productivity and the aggregate capital stock. Then  $K'^{en} < K'$  and  $\mathbb{E}[z'|\mu_{t|t}^z] < \mathbb{E}[z'|z]$  implies that when the economy booms, potential entrants underestimate future equilibrium wages and overestimate the future value of final goods. As a result, even though we set  $\rho_z = \rho_x$ , by misunderstanding general equilibrium forces, potential entrants overestimate their expected profit from entry. Conversely, when economy falls into recession the opposite occurs. In this way, the informational friction imposed on potential entrants amplifies fluctuations in entry along the aggregate productivity driven business cycle.

One interesting question is what will happen to potential entrants who entered the market during a boom period after overestimating expected profit from entry. If they exit the market as soon as they realize the true aggregate status and their own idiosyncratic productivity, then the procyclical entry generated by this learning mechanism would not have a meaningful effect on labor market equilibrium or aggregate dynamics. Whether this occurs depends on the relative size of the fixed entry and operating costs. Given that the entry cost is sunk, once a potential entrant becomes an incumbent plant, they make their exit decision by comparing their expected profit from operating to the fixed operating cost. So if the entry cost is relatively large compared to the fixed operating cost then these plants who entered the market based on wrong estimations of expected profit choose to keep operating even after they learn about the true state of the economy.

In calibrating the model economy, the most relevant moments for the entry and operating

<sup>19</sup>Variables marked with ' represent period  $t+1$  values.

<sup>20</sup> $G(z'|z)$  represents the conditional probability distribution of  $z'$  given  $z$ .

costs are relative productivity and employment size of entering and exiting plants. If entry costs are actually higher than operating costs then we should observe from the data that entering plants have higher productivity and large employments compared to the average exiting plant. This is what we actually observe in the data. The average entering plant has higher TFPR and more workers than the average exiting plant. Therefore because of the gap between entry and operating costs, not all plants who entered the market based on incorrect estimations of expected profits choose to exit even after they learn the true state of the economy.

As the boom-induced entry cohort grows, they generate additional labor demand. Because of the additional labor demand associated with these new plants, equilibrium wages stay higher for some time than the aggregate productivity level would normally suggest. For incumbent plants whose expected profit is low - those close to the exit margin - the additional labor demand generated by boom cohorts pushes some into ceasing operations. This makes exit roughly acyclical. To make clear that this point depends on the informational structure, we will also provide results from the version of the model economy where potential entrants have full information just as incumbent plants do. In this case, potential entrants correctly perceive expected profit, resulting in less entry, and consequently less labor demand and less exit in booms, yielding a less procyclical entry margin and a fully countercyclical exit margin.

Another critical issue concerns why potential entrants cannot fully infer aggregate productivity from observing the current period equilibrium wage. Imagine that at time  $t$  the economy is hit by a positive aggregate productivity shock. Due to the factor adjustment costs, labor demand responds sluggishly to the positive shock. Since the wage is the most important source of information for a potential entrant, the wage response being relatively muted on impact of the positive shock leads potential entrants to underestimate the increase in aggregate productivity. Given the persistence of the aggregate shock, potential entrants also underestimate future aggregate productivity and wages, and overestimate future idiosyncratic productivity. As a result, the number of potential entrants who decide to enter is larger compared to the full information model.

As we have discussed earlier, most plants who enter the market based on an incorrect estimation do not immediately exit. Some of them survive and generate additional labor

demand as they grow. Now return to the time  $t$  incumbent's labor demand decision. Because incumbents can exactly observe the aggregate state, they can accurately expect that from next period on ( $t + 1, t + 2, \dots$ ) wages will be high compared to aggregate productivity. This means that incumbents know going forward with too much labor is particularly costly due to the adjustment cost. The forward looking nature of the incumbent's labor demand decision dampens the response in labor demand and consequently dampens the response of the equilibrium wage. As a result current potential entrants observing the wage further underestimate the aggregate productivity increase and further overestimate their expected profit from entry.

In sum, compared to aggregate productivity, time  $t$  wages will be low relative to the shock and time  $t + 1, t + 2, \dots$  wages will be high relative to the shock. Equilibrium wage dynamics will be hump-shaped and their dynamics do not closely follow the dynamics of aggregate productivity. If potential entrants could get information about aggregate productivity from all relevant moments of the distribution of incumbents and potential entrants jointly with the equilibrium wage, then they would be able to form precise estimates for aggregate productivity. But as long as potential entrants are boundedly rational and using only information from a limited set of moments (specifically, only the mean of the incumbent plants' distribution over capital), they cannot fully recover aggregate productivity from observing current period equilibrium wages.

## 4. Calibration

The model period is annual. We used a time discount rate ( $\beta$ ) of 0.9615 so that the average annual implied interest rate is approximately 4%. The depreciation rate of capital ( $\delta$ ) is set at 6.5%, in the middle of the range that has been used in the prior literature discussing micro level plant behavior in the manufacturing sector.<sup>21</sup> We set the returns to scale parameter ( $\theta$ ) as 0.805 - close to the lower end of the estimates of the manufacturing sector's returns to scale from [Lee \(2005\)](#). The diminishing returns on capital in the production function is captured by setting  $\alpha$  as 0.3. According to [Atkeson and Kehoe \(2005\)](#), the physical capital share in manufacturing is 19.9% and the intangible capital income share is 8%. Given that our model

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<sup>21</sup>For example, [Atkeson and Kehoe \(2005\)](#) used 5.5% and [Cooper and Haltiwanger \(2006\)](#) used a depreciation rate of 6.9%.

economy does not have explicit intangible capital, we attribute half of the intangible capital share to the physical capital share and match this at 24%. Risk aversion and the Frisch elasticity of labor supply are set as 1 and 1.5 respectively.

Table 3: Parameter values

Description	Symbol	Value
Time Discount	$\beta$	0.9615
Capital Share	$\alpha$	0.3
Return to Scale	$\theta$	0.805
Depreciation Rate	$\delta$	6.5%
Risk Aversion	$\sigma_c$	1
Frisch Elasticity	$\sigma_n$	1.5

In the model economy, the only aggregate shock is the aggregate productivity shock. So we try to match the cyclical behavior of the *model economy's aggregate output* with the cyclical behavior of the *manufacturing sector's real output that is explained by productivity shocks*. To construct this in the data, we projected HP filtered real manufacturing output (BLS annual data) on three lags of HP filtered manufacturing sector TFP in levels (constructed from NBER CES data), plus quadratic and cubic terms. We then calibrated the persistence and volatility of the aggregate shock process by targeting the standard deviation and AR(1) coefficient of the fitted output series.

Table 4: Cyclical properties of output

	S.D. (%)	$\rho$
Data(fitted $y$ )	3.7	0.45
Baseline	3.6	0.53
Full Info.	3.3	0.4

In the following quantitative exercise, to highlight the role of the information friction, we compare the business cycle behavior of our baseline model with a ‘full information model’ wherein potential entrants can separately observe aggregate productivity and their potential idiosyncratic productivity, but is otherwise identical to the baseline model. Ideally, both models would be able to capture the true persistence of output to ensure the best comparison.

But in this case the difference between the two models is relatively large, so for this exercise we calibrated so that the midpoint of the two models is consistent with the data target.

Table 5: Shock processes

Description	Symbol	Value
S.D. of innovation to aggregate productivity	$\sigma_z$	0.025
Persistence of aggregate productivity	$\rho_z$	0.68
S.D. of innovation to idiosyncratic productivity	$\sigma_x$	0.15
Persistence of idiosyncratic productivity	$\rho_x (= \rho_q)$	0.68

As already explained in Section 3.5, to prevent the information friction from mechanically amplifying entry, we choose the persistence of incumbents' idiosyncratic shock process and potential entrants' potential idiosyncratic shock process to be consistent with the persistence of the aggregate shock process. Our choice for the persistence of the idiosyncratic shock process falls into the middle of the range of estimates in the literature. Regarding the volatility of the idiosyncratic shock process, what matters for potential entrants' learning problem is the relative variance between innovations to aggregate productivity and *potential idiosyncratic productivity*. But, estimates of the volatility of the idiosyncratic shock process in the literature are for incumbent plants. Therefore, we only choose the volatility of the idiosyncratic shock process of incumbent plants from estimates in the literature. We pin down the volatility of the potential entrants' potential idiosyncratic shock process through an endogenous calibration procedure that will be discussed in the next subsection.

Given that we will include entering plants' employment size and productivity relative to those of continuing plants as target moments in this endogenous calibration procedure, there is tendency that as incumbent plants' idiosyncratic shock process becomes more volatile so does the volatility of the potential idiosyncratic shock process. In that sense, we make a conservative choice regarding the volatility of the idiosyncratic shock process of incumbent plants. We choose  $\sigma_x = 0.15$  which is on the lower end of estimates in the literature.<sup>22</sup>

We match moments generated from the steady state version of the model economy where there is no aggregate shock with empirical moments reported in the literature<sup>23</sup> based on the

<sup>22</sup>See, Abraham and White (2006), Foster et al. (2008), and Castro et al. (2015).

<sup>23</sup>Moments related to entry and exit behavior are reported in Lee and Mukoyama (2014). Plant level

Table 6: Existing estimates of idiosyncratic shock persistence

Source	$\rho_x$
<a href="#">Cooper and Haltiwanger (2006)</a>	0.92
<a href="#">Foster et al. (2008)</a>	0.757 $\sim$ 0.814
<a href="#">Castro et al. (2015)</a>	0.439
<a href="#">Lee and Mukoyama (2015c)</a>	0.969

LRD (Longitudinal Research Database). One advantage of relying on LRD data instead of BDS data (Business Dynamics Statistics) is that the LRD provides information on both the relative employment size and productivity (which is not available in the BDS) of entering and exiting plants. Another reason is that plant level investment rates are constructed from the LRD.

The first set of moments to be calibrated are the entry rate, exit rate, relative (to continuing plants) employment size and productivity of entering plants and relative (to continuing plants) employment size and productivity of exiting plants. Between two successive periods  $t - 1$  and  $t$ , exiting plants are defined as those who operated at time  $t - 1$  but do not operate at  $t$ , while continuing plants are those who operate during both  $t - 1$  and  $t$ . Entering plants are those who start operating at  $t$ . Relative characteristics are constructed in a consistent way as in [Lee and Mukoyama \(2015a\)](#). That is, we used time  $t$  characteristics of entering and continuing plants and time  $t - 1$  characteristics of exiting plants when we calculate relative employment size or productivity. Entry and exit rates are calculated as a measure of entering or exiting plants compared to the average total measure of plants between period  $t - 1$  and  $t$ . Because the entry rate and exit rate are identical in the steady state of the model economy, we target the midpoint of the entry and exit rates from the data.

The second set of moments calibrated using model data are the average plant level investment rate ( $\frac{i}{k}$ ), the fraction of plants whose investment rate is higher than 20%, and the fraction of plants whose investment rate is less than  $-20\%$ . These moments are mainly related to the adjustment cost of capital. One issue is that Cooper and Haltiwanger constructed these statistics from a balanced panel without entry and exit margins. To be consistent with

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investment rate related statistics are available from [Cooper and Haltiwanger \(2006\)](#) and job flow data are available at the webpage of John Haltiwanger.

the data, we calculated these moments in the model economy using only continuing plants.

We disciplined the magnitude of the linear adjustment cost of labor by matching gross job flows in the manufacturing sector. Specifically, we matched the sum of the job creation rate from entering and continuing plants and the job destruction rate from exiting and continuing plants. There is one issue in matching total flow rates. We already targeted the entry rate and the relative average employment size of entering plants. Targeting these two moments pins down the job creation rate from entering plants which is higher than the corresponding statistics from job flows data. To prevent our model economy from consistently overestimating general equilibrium pressure generated from entering plants, we also magnify job flows from continuing plants. So rather than matching a total job flow rate of around 19% in the job flows data from the webpage of John Haltiwanger, we target a somewhat magnified 24% total job flow rate.

We finally calibrate so one third of the available time of the representative household is spent on market work.

The complete set of model parameters we used to jointly match all the just mentioned target moments are as follows: The entry cost, fixed operating cost, stochastic process for the potential idiosyncratic process (long-run mean and volatility of the process), capital stock for entering plants, capital adjustment cost, labor adjustment cost, fixed measure of potential entrants, and scale of disutility from work. We specified the stochastic process for the fixed operating cost as a log-normal distribution.<sup>24</sup>

The model economy has strong implications for the life cycles of plants. From the BDS, (Business Dynamics Statistics)<sup>25</sup> we can construct both age dependent average employment sizes and employment shares. We then compare the steady state of the model economy's (which is used for the calibration) implied life cycle pattern and those constructed from the BDS. Recall that not the whole age profile is the calibration target. We only targeted the relative employment size and productivity of entering and exiting plants. The gap in productivity between entering plants and continuing plants together with the mean reverting

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<sup>24</sup>Given that our choice of the log-normal distribution is somewhat arbitrary we will provide robustness checks using the Pareto distribution in the appendix. Results from the model economy following Pareto innovations are almost identical to those from the model economy using a log-normal distribution.

<sup>25</sup>We have not found detailed age dependent average employment size statistics constructed using the LRD.

Table 7: Parameter values associated with matching moments

Description	Symbol	Value
Entry Cost	$c_e$	0.02
Operating Cost(Mean)	$\mu_{cf}$	-4.68
Operating Cost(S.D.)	$\sigma_{cf}$	1.38
Mean of Potential Idio.	$\bar{q}$	-0.41
S.D. of Potential Idio.	$\sigma_q$	0.11
Entrant's Capital	$k_{en}$	0.15
Resale Value of Capital	$p_s$	0.91
Labor Adj. Cost-Hiring	$c_p$	0.23
Labor Adj. Cost-Firing	$c_n$	0.20
Measure of Entrants	$M$	27.52
Disutility from work	$\gamma$	5.08

Table 8: Target moments

Moment	Target	Model
Entry Rate	6.1%	6.1%
Exit Rate	6.1%	6.1%
Relative Size of Entering Plants	56%	56%
Relative Size of Exiting Plants	46%	46%
Relative Prod. of Entering Plants	94%	94%
Relative Prod. of Exiting Plants	85%	85%
Avg. $\frac{i}{k}$	12.2%	9.4%
Positive Spike( $\frac{i}{k} > 0.2$ )	18.6%	16.2%
Negative Spike( $\frac{i}{k} < -0.2$ )	1.8%	2.9%
Hours	33%	33%
Job Reallocation Rate	24%	24%

property of the incumbent's idiosyncratic shock process generates an increasing age profile in idiosyncratic productivity. The age profiles of average employment size and employment share<sup>26</sup> in the model economy are mainly a combination of the age profile of idiosyncratic productivity and the presence of factor adjustment costs.

Another way to understand this exercise is as an answer to the following: given the observed age profiles of average employment size and employment share in the data, how

<sup>26</sup>The employment share of age  $j$  plants is defined as  $\frac{\text{Employment at age } j \text{ plants}}{\text{Total employment}}$ .

Figure 4: Model Implied Average Idiosyncratic Productivity by Plant Age

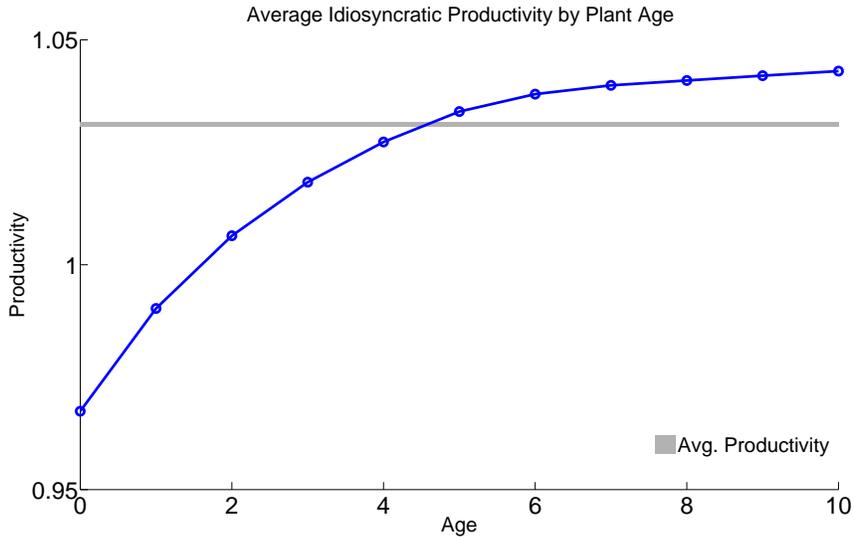


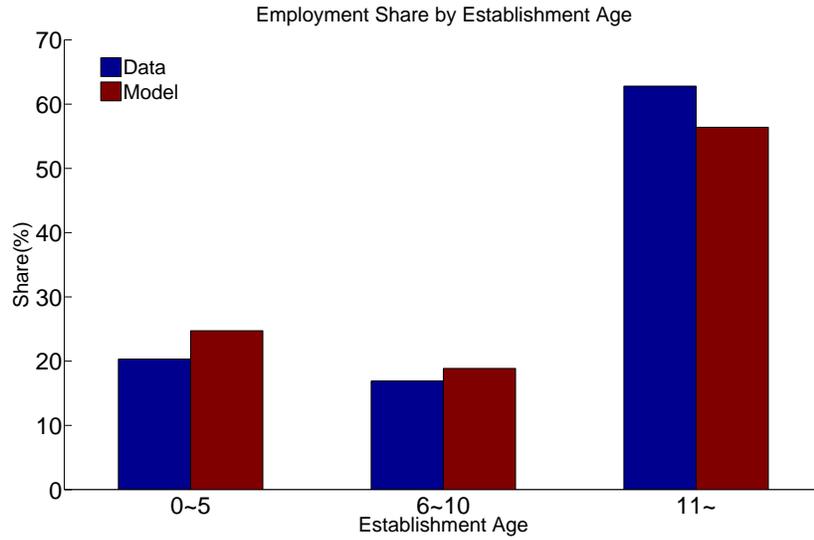
Figure 5: Average employment size by plant age



much of them can be attributed to the combination of initial characteristics of entering plants and factor adjustment costs?

Figure 4 and 5 show the age profiles for average idiosyncratic productivity and average employment size respectively. Average idiosyncratic productivity grows as a concave function of age. But in the average employment size profile, the concave pattern is not as transparent as it was for the productivity profile. This is because the presence of both capital and labor adjustment costs defers plants from growing enough to catch up to the step productivity

Figure 6: Employment share by broad plant age



profile during infancy. The relatively smooth growing pattern of average employment size at ages between 0 ~ 5 is quite consistent with what is observed from the BDS.<sup>27</sup>

In terms of how the age composition of the economy affects labor market equilibrium, what eventually matters is how employment is spread across plants of different ages. Figure 6 shows employment share distribution by plant age. One issue here is that we calibrated the model economy using both the entry rate of new plants and the relative employment size of entering plants to continuing plants from the LRD as target moments. This means that the model employment share of age 0 plants is pinned down by the corresponding statistics from the LRD.

However, the employment share of age 0 plants is not identical in both data sets. So we multiplied the BDS-based employment shares from ages 0 to 10 as follows:

$$\text{age } i \text{ employment share from BDS} \times \frac{\text{age 0 share from LRD}}{\text{age 0 share from BDS}}, \text{ for } i = 0, 1, \dots, 10$$

to place the model on the same scale as the data we compare it to. In Figure 6, the employment share from the data is this modified labor share: the hypothetical employment shares in the LRD that would arise if the age 0 ~ 10 employment shares profile were identical between the LRD and BDS.

<sup>27</sup>Because the publicly available portion of the BDS only provides pooled data between age 6 ~ 10, we only compared employment size profiles at ages between 0 ~ 5.

In the model economy, both age 0  $\sim$  5 and age 6  $\sim$  10 plants have a higher share than what we can observe from the data. Given that the model economy has generated quite similar average employment size profiles between age 0  $\sim$  5, the mismatch of the employment shares across ages 0  $\sim$  5 implies that the real exit profile is steeper at age 0  $\sim$  5 than the exit profile in the model economy. Despite the slight mismatch, the age shares of young plants are still largely consistent between the model and the data.

## 5. Results

### 5.1. Impulse Response Exercise

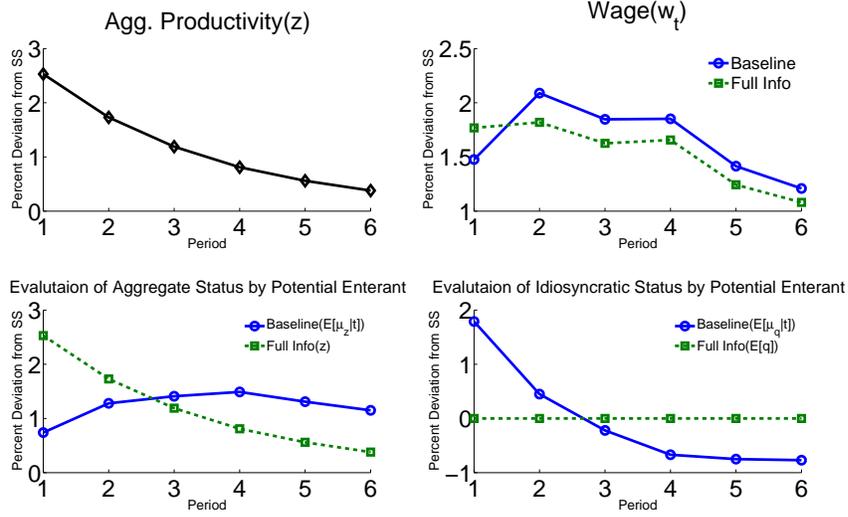
To understand the model economy’s dynamics and especially the role of the information friction faced by potential entrants, we provide impulse response functions from both the ‘baseline model’ and the ‘full information model’. In the baseline economy, potential entrants cannot separately observe the aggregate and potential idiosyncratic productivity shocks, while in the full information model potential entrants can observe aggregate productivity directly. The only difference across the two economies is this difference in the information set of potential entrants. Therefore, any different dynamic responses generated via a positive aggregate productivity shock will be solely attributable to the different information structure of potential entrants.

In Figure 7, we plot the dynamics of aggregate productivity, equilibrium wages, and the average of potential entrants’ estimates about both aggregate and idiosyncratic productivity. At time 0 the economy is at its steady state.<sup>28</sup> At period 1, the economy is hit by a one unit positive impulse in aggregate productivity. Recall that in the baseline model, potential entrants learn about the aggregate state from some exogenous signal ( $z_t + q_t$ ), current period equilibrium wages ( $w_t$ ), and the aggregate capital stock ( $K_t$ ). As we have already seen from Figure 3, potential entrants attribute most of the fluctuations in  $z_t + q_t$  to the idiosyncratic component ( $q_t$ ) and aggregate capital stock movements to lagged fluctuations in  $z_t$ .

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<sup>28</sup>One caution is that the concept of the steady state used in the impulse response exercise is different from that used in the calibration. The steady state used in the calibration is the version of the economy where there is no aggregate shock. But the steady state used as an initial period in the impulse response exercise is where there is an aggregate shock and all agents account for equilibrium price dynamics when they form expectation with respect to the aggregate state. But, the actual realization of the aggregate shock is fixed at its mean level for 100 years.

Figure 7: Impulse response of wages and beliefs

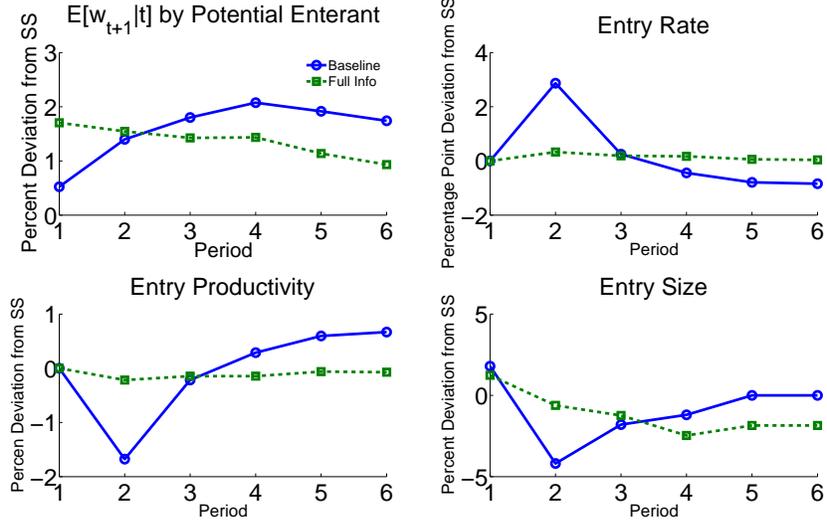


Therefore, on impact of the aggregate productivity shock, variation in the current period equilibrium wage is the most important source of learning for potential entrants. But on impact of the shock, the response of the equilibrium wage is somewhat dampened (adjustment costs) and as a result potential entrants under-estimate the fluctuations in  $z_t$ . In the third and fourth panel of the Figure 7, we plot dynamics of  $\int \mu_{t|t}^z(z_t + q_t, \mu_{t|t-1}^z, \mu_{t|t-1}^q; w_t, K_t) d\Omega(z + q, \mu^z, \mu^q)$  and  $\int \mu_{t|t}^q(z_t + q_t, \mu_{t|t-1}^z, \mu_{t|t-1}^q; w_t, K_t) d\Omega(z + q, \mu^z, \mu^q)$  respectively. In words, the average estimates of aggregate productivity and potential idiosyncratic productivity across potential entrants at the point in period  $t$  when they have to make their entry decision. Until period 2, potential entrants underestimate fluctuations in  $z_t$ . But because equilibrium wages fall slowly compared to aggregate productivity, and because the aggregate capital stock adjusts slowly in the presence of adjustment costs, from period 3 on potential entrants overestimate  $z_t$ . Given that potential entrants can observe  $z_t + q_t$ , estimates of  $q_t$  move inversely with estimates of  $z_t$ .

In the first panel of Figure 8, we plot the average one period ahead wage forecast across potential entrants.<sup>29</sup> As potential entrants underestimate  $z_t$  until period 2, they also underestimate  $E_{en}[w_{t+1}|t]$  until period 2 and consequently overestimate expected profit from entry

<sup>29</sup>In the baseline economy, potential entrants are aware of the equilibrium price forecasting rule and the law of motion of the aggregate capital stock, as are incumbent plants, but they cannot observe  $z_t$ . Rather they use posterior estimates of  $z_t$  when forming expectations for tomorrow's wage. Given that each potential entrant has a different posterior estimate for  $z_t$ , each also has a different wage forecast.

Figure 8: Impulse responses of wage forecasts and the entry rate



until period 2. Since at period 1,  $E_{en}[z_2 + x_2|t]$  is much higher than  $E_{en}[w_2|t]$ , the number of potential entrants who decide to enter increases much more in the baseline model compared to the full information model. As a result, a large number of new plants start operating in period 2 and the entry rate<sup>30</sup> in period 2 responds by more in the baseline economy.

Another interesting feature of the entry rate dynamics in the baseline model is that the entry rate peaks in period 2 and then declines quickly. This happens because in the baseline economy potential entrants learn about the aggregate state mainly from the dynamics of the equilibrium wage and aggregate capital stock. As both of these move slowly due to adjustment costs, early in the boom potential entrants under-estimate aggregate productivity and later they over-estimate aggregate productivity as the shocks dissipate.

The dynamics of the entry rate in the baseline model naturally follow. In the full information model, because potential entrants can accurately form expectations over the equilibrium wage dynamics associated with the positive aggregate productivity shock, the response in the entry rate is quite dampened and it smoothly declines following the aggregate productivity dissipation. This pattern of entry rate dynamics in the baseline model is consistent with the empirical VAR exercise (Recall Figure 1 in Section 2).

Conversely, the dynamics of the characteristics of entering plants mirror the dynamics of

<sup>30</sup>We defined the entry rate at period  $t$  as  $\frac{\text{measure of plants that start operating at period } t}{\text{average total number of plants between period } t-1 \text{ and } t}$  which is consistent with how BDS measures the entry rate.

Figure 9: Dynamics of Age Composition

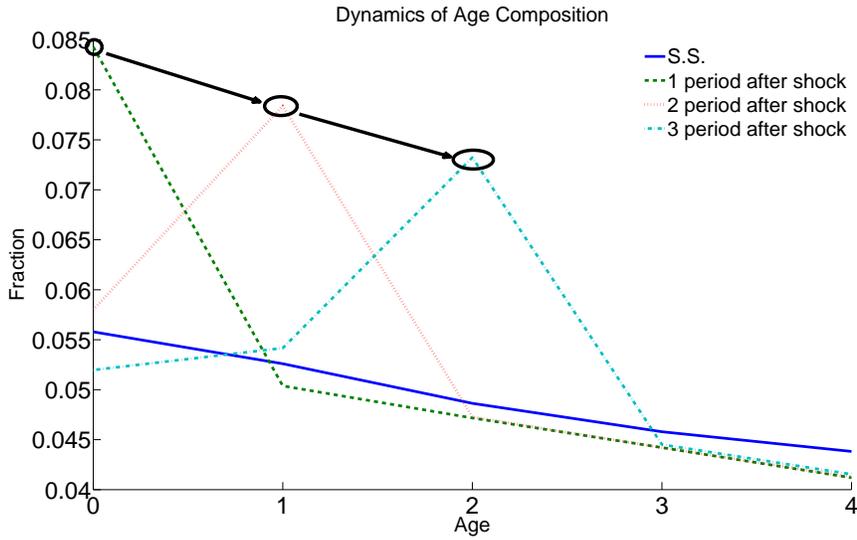
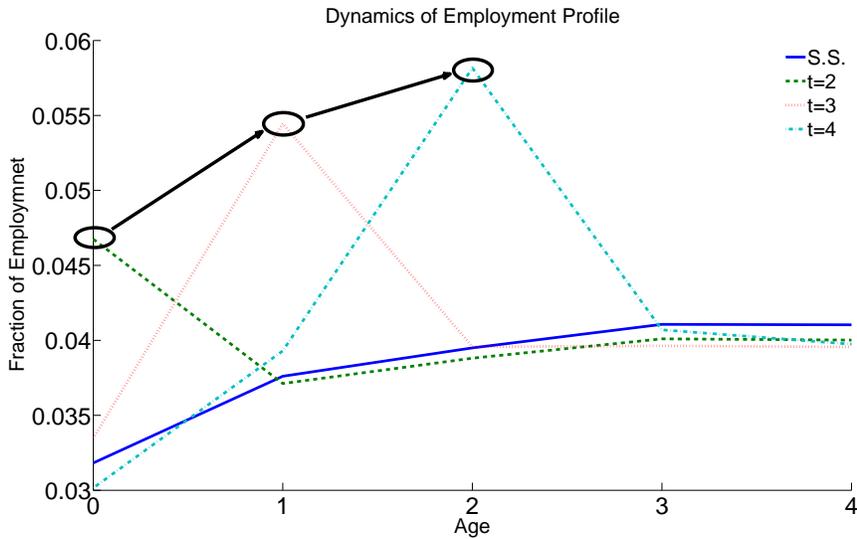


Figure 10: Dynamics of Employment Share Profile



the entry rate. In period 1 of the baseline model, as more of the potential entrants decide to enter based on their overestimation of the expected profit from the entry, average productivity and the average employment size of the entering plants deteriorates. This pattern is also consistent with the data. According to Lee and Mukoyama (2015a), plants that enter during recessions have significantly better TFPR than plants that enter during booms.

Regarding the issue of whether potential entrants' information friction has an effect on the aggregate dynamics, the important question is what will happen to those plants who

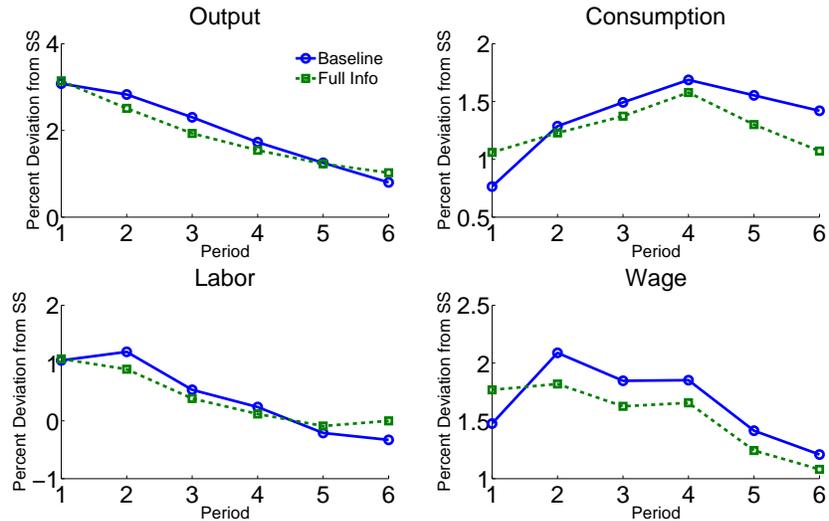
entered the market based on an incorrect estimation of the aggregate state. To see what happens to the cohort who entered the market at period 2, in Figure 9 and Figure 10 we plot how the age composition and employment share profiles change after the positive aggregate productivity shock in the baseline model. In both figures, “1 period after the shock” indicates period 2 in the impulse response exercise. To track the cohort who entered the market in period 2 we track the measure of age 0 plants “1 period after the shock”, the measure of age 1 plants “2 period after the shock” and the measure of age 2 plants “3 period after the shock”.

What we can see is that most of the plants who entered the market in period 2 do not exit the market immediately. Even if many entered due to a poor estimation of the expected profit from entry, not all immediately exit the market after they realize the true aggregate state. The reason for their continuing operation is that the entry cost is relatively larger than the fixed operating cost. Given that the entry cost is sunk, once potential entrants become incumbent plants, what matters is whether the expected profit from operating is larger than the stochastic operating fixed cost.

To see the effect of this period 2 entering cohort on labor market equilibrium, the key is in how the age profile of the employment share changes after the mass entry. From Figure 10 we can see that as the period 2 entrants grow in size as they age, they account for a larger share of total employment and generate additional wage pressure. This additional labor demand associated with the cohort that entered the market in period 2 keeps the equilibrium wage from declining as aggregate productivity declines.

The dynamics of the age and employment shares affects aggregate dynamics. In Figure 11, we plot dynamics for aggregate output, consumption, labor, and wages. In the baseline economy, because of the strong response along the entry margin, all aggregate variables decline slowly compared to in the full information economy. The information friction faced by potential entrants amplifies fluctuations in the entry margin, which acts as an effective internal propagation mechanism. Particularly, aggregate labor and wages show a clear hump shaped response in the baseline model. This is caused by the interaction of the incumbent plants’ forward-looking behavior with respect to labor demand and the additional labor demand generated from the shock-induced entry. In contrast to potential entrants, incumbent plants can directly observe aggregate productivity  $z_t$  and as a result they can properly (con-

Figure 11: Impulse Response of Aggregate Variables

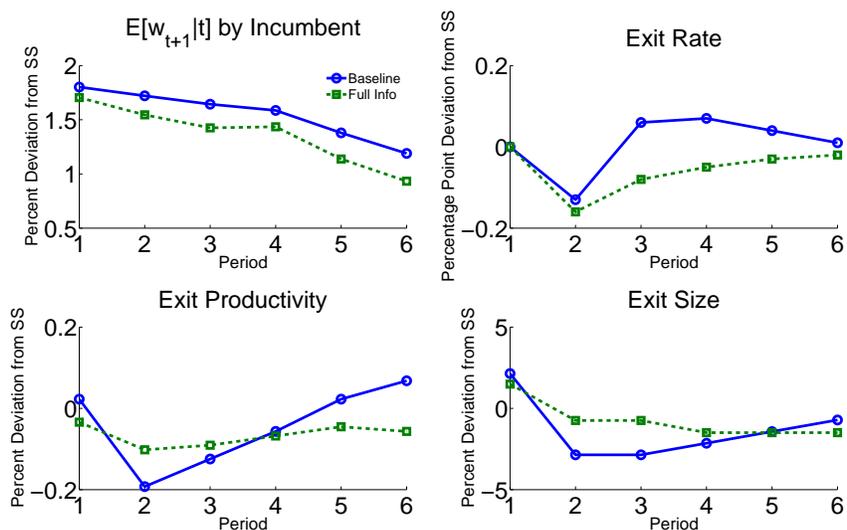


sistently with actual wage dynamics) forecast next period’s wages. They correctly expect that after period 2, equilibrium wages will decline more slowly than aggregate productivity. In the presence of the labor adjustment cost, higher expected future wages reduce labor demand from current incumbents.

More formally, these dynamics result because when  $E[w_2|1]$  is relatively higher than  $E[z_2|1]$ , period 2’s static optimal labor demand is on average lower than period 1’s static optimal labor demand. So if incumbents in period 1 choose to employ too much labor then they must pay adjustment costs to reduce their labor in period 2. By accounting for their future labor decisions, the period 1 response of incumbent labor demand to the increase in aggregate productivity is lessened. With less aggregate labor demand in period 1, the equilibrium wage in period 1 is also lower and potential entrants, by observing the slight increase in  $w_1$  relative to  $w_0$ , underestimate  $z_1$ . Then entry in period 2 responds strongly and as the entry cohort grows, aggregate economic activity declines slowly relative to the decay of aggregate productivity. Recall Figure 7, where we generated an impulse response from how potential entrants evaluate aggregate productivity by observing the dynamics of the equilibrium wage. In Figure 11, we observe that as potential entrants learn about the aggregate state from the dynamics of the equilibrium wage and make entry decisions based on the result of their learning, they drive how the labor market unfolds going forward.

In the first panel of Figure 12, we plot the dynamics of incumbent plants’ one period ahead

Figure 12: Impulse response of the exit rate



wage forecast. We can see that compared to the full information model, here incumbent wage forecasts decline slowly. Given that we defined the exit rate in period  $t$  as

$$exit\ rate = \frac{\text{measure of plants who exit between period } t - 1 \text{ and } t}{\text{average total number of plants between period } t - 1 \text{ and } t}$$

the period  $t$  exit rate is mainly determined by the exit decisions of  $t - 1$  incumbent plants. Therefore, the period  $t$  exit rate critically hinges on the wage forecasts of period  $t - 1$  incumbents,  $E[w_t|t - 1]$ . In period 2, the additional labor demand generated by the cohort who entered in response to the shock<sup>31</sup> is not large enough compared to the level of the aggregate productivity ( $z_2$ ) to significantly affect exit.

In other words, from the perspective of the period 1 incumbents,  $E[w_2|1]$  is not particularly large compared to  $E[z_2|1]$ . As a result, the number of plants who decide to exit the market decreases and the period 2 exit rate falls below the steady state exit rate. But as time goes by, the additional labor demand generated by the growing boom cohort outgrows aggregate productivity, pushing the exit rate from period 3 on above the steady state level. This oscillation in the exit rate implies that the correlation between exit and aggregate output will be very low. Conversely, in the full information economy, the exit rate decreases significantly in period 2 and then as aggregate productivity declines it returns to its steady state level.

<sup>31</sup>See age 0 plants' employment share in the "1 period after the shock" profile of Figure 10.

## 5.2. The Business Cycle Behavior of Entry and Exit

Using the time series generated from stochastic simulation of the model economy, we can test which model economy generates cyclical behavior in entry and exit that is more consistent with the data. For the stochastic simulation, we used the same randomly generated sequence for aggregate productivity ( $\{z_t\}$ ) in both the baseline and full information economy. Table 9 presents the contemporaneous correlation of HP filtered output with either the HP filtered entry rate or the HP filtered exit rate. Both of the model economies imply that the entry rate comoves positively with output, which is consistent with the data. But the full information model implies a countercyclical exit rate that is at odds with the data. As we have seen from Figure 12, the baseline model generates an oscillating aggregate exit rate, which gives a realistic correlation between output and the exit rate that is close to 0.

Table 9: Correlation with output

	Entry Rate	Exit Rate
Data	0.35	-0.02
Baseline	0.49	-0.09
Full Info	0.5	-0.49
Asy. Cost	0.46	-0.47

Note: In the data, the reported correlations are between the entry (exit) rate from BDS and industrial production in the manufacturing sector. In the model economy, correlations are between the entry (exit) rate and aggregate output. All correlations are calculated using HP filtered series.

Another important aspect of the cyclical behavior of entry and exit is the magnitude of the fluctuations. To see this dimension of the cyclical behavior, following Lee and Mukoyama (2015a), we categorize each period in the model economy: as a boom if output growth is higher than average and as a recession if output growth is lower than average. Table 10 provides the average entry and exit rates across all boom and recession periods. The baseline model generates both large fluctuations along the entry margin and an acyclical exit margin. Less realistically, the full information economy generates overly weak entry and overly strong countercyclical exit fluctuations.

The cyclical behavior from the full information model economy has a critical implication

Table 10: Magnitude of Fluctuations

	Entry Rate(%)		Exit Rate(%)	
	Boom	Rec.	Boom	Rec.
Data	8.1	3.4	5.8	5.1
Baseline	7.6	4.5	5.8	5.8
Full Info	6.3	5.9	6.0	6.2
Asy. Cost	6.0	5.9	5.9	5.9

Note: The data column is taken from [Lee and Mukoyama \(2015a\)](#). In the model economy, booms and recessions are categorized depending on whether output growth rate is above or below the average output growth rate as in [Lee and Mukoyama \(2015a\)](#).

regarding the recent literature that extends [Hopenhayn \(1992\)](#) with business cycle fluctuations.<sup>32</sup> Once we impose goods market clearing, a labor market clearing condition with proper general equilibrium forces (realistic risk aversion and Frisch elasticity), and adjustment frictions both for capital and labor, then it is hard to match the fluctuations in entry and exit observed in the data.

Then what can explain the observed fluctuations in the entry margin? To be consistent with the data, a potential mechanism should be able to amplify the fluctuations along the entry margin and the generated exit should be simultaneously acyclic. But under a full information structure, as shown, this is far from trivial. Because both the potential entrant's incentive to enter and the incumbent's incentive to keep operating depend on the incumbent's expected value, under the symmetric information structure, exit is just as countercyclical as the entry rate is procyclical. However, the baseline model with incomplete information and learning can generate both a procyclical entry rate and an acyclical exit rate at the same time, by cutting this tight linkage between perceived expected profit for the potential entrant and incumbent. In the baseline economy, the exit rate is acyclical because entry is too strongly procyclical.<sup>33</sup>

To highlight the unique role of the asymmetric information structure of potential entrants and incumbent plants in generating asymmetric cyclical behavior of entry and exit, we modify

<sup>32</sup>For example, [Clementi and Palazzo \(2015\)](#), [Clementi et al. \(2015\)](#).

<sup>33</sup>This implication of the baseline model economy is reminiscent of the result from [Caballero and Hammour \(1994\)](#). Inside a model of creative destruction, they showed that in response to the aggregate demand fluctuation there is a trade-off between the cyclicity of the creation and destruction margins.

the full information economy such that the fixed cost of operating is paid in terms of labor rather than final goods. Recall that currently in both model economies, both the entry and fixed operating costs are paid in terms of the final good. This modification changes the incumbent plants' value function from (6) to

$$V(n_{-1}, k, x; z, \Lambda) = \int \max\{V_c(n_{-1}, k, x; z, \Lambda) - w\xi, V_x(n_{-1}, k, x; z, \Lambda)\}dG(\xi) \quad (37)$$

Additionally, the labor market and goods market clearing conditions ((27) and (28)) are modified accordingly, and this modified model economy is calibrated using the same target moments as the baseline and full information models.

The cyclical behavior of entry and exit implied from this alternative economy is presented in the last row of Table 9 and Table 10, labeled as “Asymmetric Cost”. In terms of the correlation with output, exit is just as countercyclical as entry is procyclical. Regarding the magnitude of the fluctuations, compared to the full information economy, this asymmetric cost model generates an acyclical exit margin because real wages are procyclical and the fixed operating cost thus becomes procyclical as it is paid in terms of labor. But in this asymmetric cost economy, fluctuations along the entry margin are also dampened. That is because potential entrants have just as much information as incumbent plants, so they can perfectly internalize the procyclical fixed operating cost. Expected profit from entry thus becomes less procyclical because the procyclical fixed cost of operating is being subtracted. The full results from the asymmetric cost model economy make it clear why it is not trivial to generate procyclical entry and acyclical exit at the same time with a symmetric full information structure.

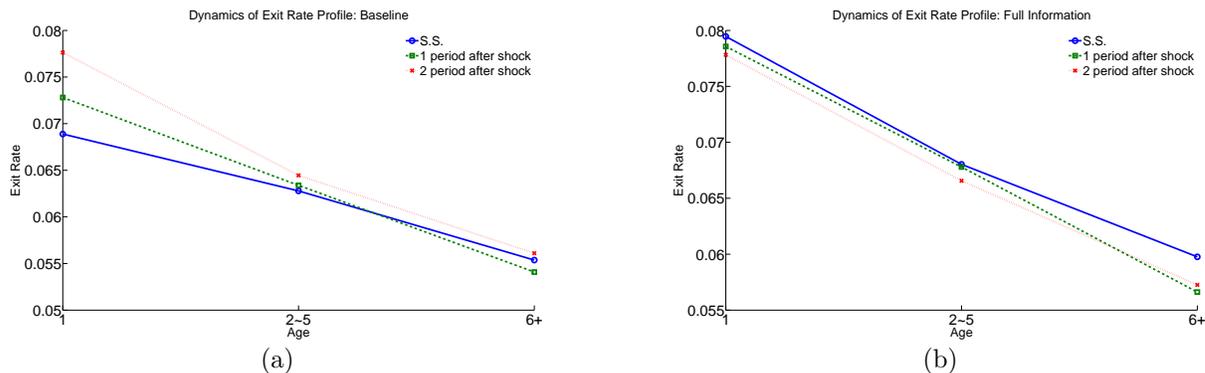
### 5.3. Exit Rate and Plant Age

Given the dynamics of the aggregate exit rate, another interesting question is which age plants exit? Because we can calculate age exit rates as a function of plant age along the business cycle from BDS, the model implied age dependent exit rate dynamics are another potential testable implication. In Figure 12, we plot how the age profile of the exit rate changes after the positive aggregate productivity shock.<sup>34</sup>

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<sup>34</sup>As mentioned already, the steady state concept used in the impulse response exercise is that of a steady state where there is an aggregate shock and agents in the model economy take into account price dynamics

Figure 13: Dynamics of the exit rate profile



Each age group’s exit rate is defined just as the aggregate exit rate was defined. For example, the exit rate of the age-1 plants in the “one period after the shock (i.e. period 2)” is determined by the exit decision of age-0 incumbents in period 1. As already mentioned, in period 1, because incumbent plants’  $E[w_2|1]$  is not large compared to  $E[z_2|1]$  and plants older than 6 years have relatively high idiosyncratic productivity, those older plants’ exit rate decreases compared to the steady state level. But as additional labor demand generated by the cohort who entered in period 2 outgrows aggregate productivity, 2 periods after the shock (i.e. period 3) the exit rate increases across all age groups. According to the “2 period after the shock” profile in the baseline model, not only does the exit rate of one year old plants increase compared to its steady state counterpart, but also older plants’ exit rate increases compared to the steady state counterpart.

Table 11: Correlation of the exit rate with lagged output by plant age

Plant Age	$IP_{t-1}$	$IP_{t-2}$
1	0.2	0.45**
2 ~ 5	0.39**	0.48***
6+	0.42**	0.42**

Note: Age dependent exit rate is from BDS. All correlations are calculated using HP filtered series. \*\* means correlation is significant at 5%. \*\*\* means correlation is significant at 1%.

associated with fluctuations in aggregate productivity. Because price dynamics and the law of motion for the distribution are different, in this concept of the steady state, the exit rate profile is not identical across the two economies.

In contrast, in the full information model, the exit rate decreases across all age groups compared to the corresponding steady state value. From the BDS, we can calculate the correlation of the HP filtered age dependent exit rate with the lagged HP filtered re-timed annual industrial production of the manufacturing sector. Across all age groups, exit has a positive correlation with the lagged output measure and most are statistically significant. Like the dynamics of the age profile of the exit rate in the baseline economy, in the BDS we see that one or two years after manufacturing sector output rises above trend, not only does the boom cohort exit rate increase, but exit also rises among pre-existing older plants.

## 5.4. Aggregate Implications

In the previous subsection, we have shown that the baseline model can generate cyclical behavior in entry and exit that is consistent with the data. A natural question is then what is the effect of the cyclical behavior of the net entry margin on the cyclical behavior of aggregate variables.

Table 12: Cyclical properties of aggregate output

	Output		Employment	
	S.D.(%)	AR(1)	S.D.(% to $y$ )	AR(1)
Baseline	3.6	0.53	43	0.71
Full Info	3.3	0.4	34	0.45

Table 12 provides the cyclical properties of aggregate output and employment generated from stochastic simulation. By comparing the cyclical behaviors of aggregate variables across two economies, we can see that the baseline model economy has both amplification and internal propagation effects. Another way to see the internal propagation effect of baseline model economy is to compare persistency of model generated solow residuals with persistency of exogenously given aggregate productivity shock process. From the Table 13, we can see that in case of the baseline model economy, the AR(1) coefficient of the Solow residual is more persistent than exogenously given aggregate productivity shock process.

Table 14 provides cross-correlations between manufacturing sector aggregate output and employment. In the data, the contemporaneous correlation between output and employment

Table 13: Persistency of model generated Solow Residual

	AR(1)
$z$	0.68
Baseline	0.73
Full Info	0.65

Note:  $z$  represents exogenously given aggregate shock process.

Table 14: Cross correlation between employment and output ( $y_t$ )

	$n_t$	$n_{t+1}$	$n_{t+2}$
Data	0.89	0.69	0.13
Baseline	0.91	0.56	0.05
Full Info	0.96	0.33	-0.17

Note: In the data, output and employment are BLS manufacturing sector annual real output and employment series. All correlations are calculated using HP filtered series.

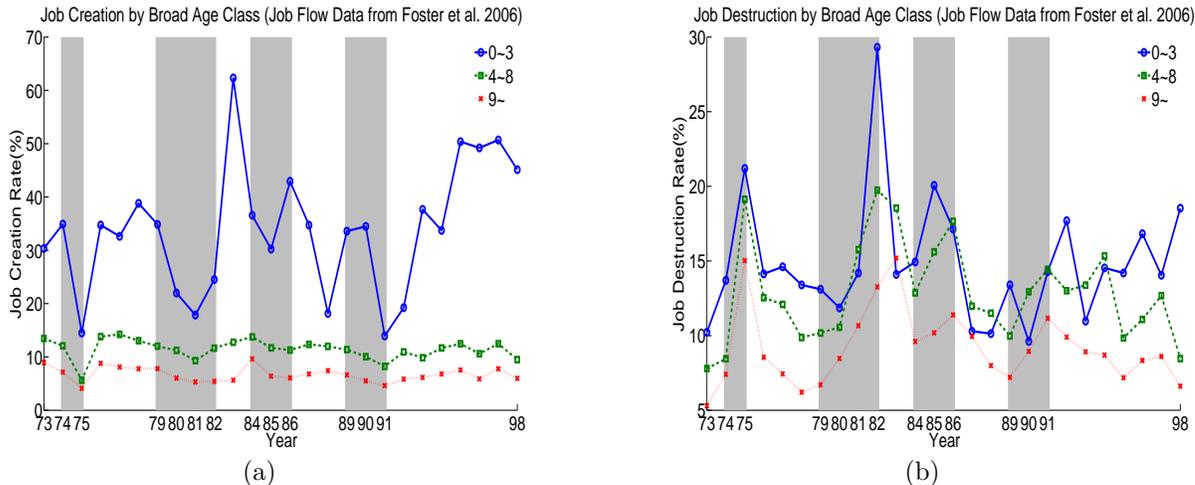
is much less than 1 but the correlation of employment with lagged output declines slowly. The corresponding statistics from the baseline economy are more consistent with the data than those from the full information model economy. This happens because due to the presence of factor adjustment costs, protracted aggregate labor demand driven by the growth of shock-induced entry cohorts dampens the response of incumbent plants to the aggregate productivity fluctuation.

## 5.5. Job Flows and Plant Age

Another testable implication of the model economy is the cyclical behavior of age dependent job flows. [Foster et al. \(2006\)](#) provides manufacturing sector gross job flows as a function of establishment age constructed from the LRD. Figure 14 shows these time series for job destruction and creation for plants of different ages. The shaded regions represent those years when the manufacturing sector's output growth rate was below the sample average growth rate. Both job creation and job destruction among the young plants (0 ~ 3 year old plants) are much more volatile than the job flows of older plants.

Further, while for older plants the magnitude of the fluctuations in job creation and

Figure 14: Job flows by plant age in the manufacturing sector

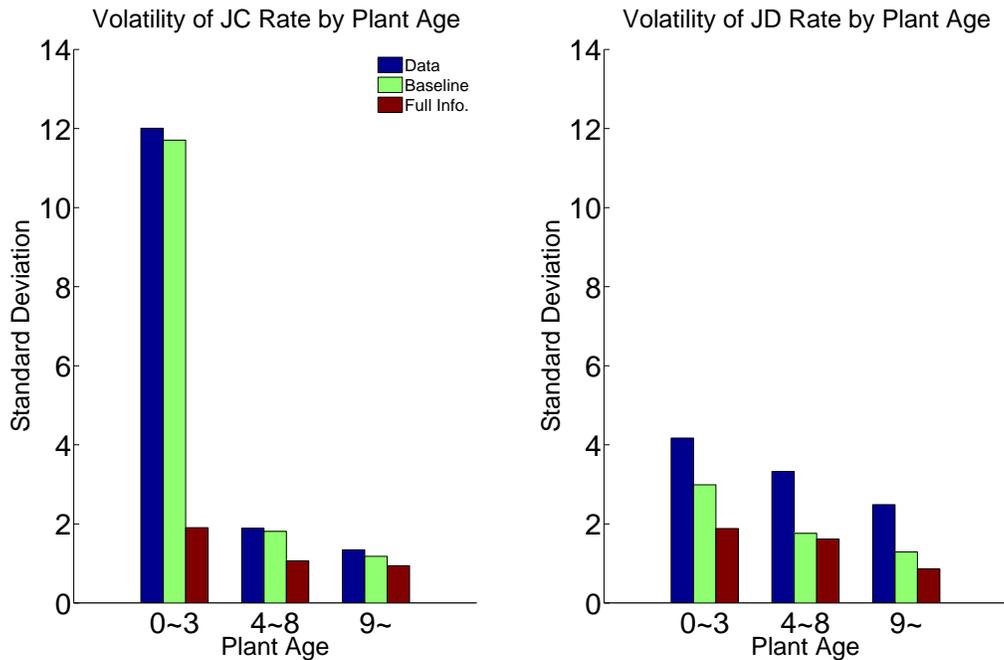


job destruction are similar, for the young plants job creation is much more volatile than job destruction. In order to check whether the model economy can generate these features observed in the data, we feed detrended actual manufacturing sector TFP from 1958 ~ 1998 into the model economy. Then, using a model generated unbalanced panel data set corresponding to the years 1973 ~ 1998, we calculate model job flows by plant age.

Figure 15 provides the standard deviations of the generated time series for job creation and destruction for each age group. The baseline model matches both the relatively large and volatile job flows of young plants and the asymmetry between the volatility of job creation and destruction also observed in young plants. The baseline economy’s ability to match the volatility of the young plants’ job creation rate mainly comes from its ability to match the volatility in entry. Additionally, as already mentioned, most of the entering plants who enter based on incorrect expectations continue operating even after they realize the true aggregate state.

This reflects that all the jobs created from young plants based on their inaccurate estimation of future profits are not quickly destroyed. As a result, in the baseline model, job destruction among young plants is less volatile than job creation over the business cycle. But, in the case of the full information economy which implies symmetric entry and exit fluctuations, young plants have a correspondingly similar volatility of job creation and job destruction.

Figure 15: Volatility of job flows by plant age



## 6. Testable Implications

Given that there is no way to explicitly observe the information set of potential entrants, we cannot directly test for the effect information frictions have on potential entrants. That said, there are testable implications of the proposed imperfect information story we can pursue. Suppose that at the enterprise level information about economic conditions, production technology, or managerial skill is shared. Then, plant entry from existing firms might be less procyclical than plant entry from new firms.

Another prediction following from an imperfect information friction is that potential entrants might overestimate their expected profit from entry during booms and underestimate profit from entry during recessions. This implies that the cohort of plants that entered during booms are more likely to exit the market relatively quickly compared to plants that entered the market during recessions. Again, this difference between boom and recession entry cohorts should be observed only for plants associated with new firms.

If potential entrants suffer from an informational friction in that they cannot disentangle aggregate and idiosyncratic productivity, then in sectors with a more volatile idiosyncratic shock the friction should have a stronger effect. According to [Castro et al. \(2015\)](#), within

the manufacturing sector there is huge variation in the volatility of idiosyncratic shocks<sup>35</sup> across sectors. Then the information friction implies that the establishment entry rate should be more procyclical in sectors with more volatile idiosyncratic shocks along the aggregate manufacturing sector's business cycle (not the disaggregated sector-specific business cycle).

## 6.1. The Cyclicalities of the Establishment Entry Rate By Firm Age

According to the BDS, among the entries that have occurred during 1977 to 2012, around 76% of entering establishments come from new firms.<sup>36</sup> The remaining 24% come from existing firms<sup>37</sup> opening new establishments, such as GM opening a new plant. If information about aggregate economic conditions or firm level productivity is shared at the enterprise level, then existing firms' decisions about establishment entry might be immune to the informational friction. But if inexperienced new firms are actually facing an information friction that makes it difficult to disentangle aggregate and idiosyncratic productivity, then establishment entry from new firms should be more procyclical than entry from existing firms.

Table 15 gives the contemporaneous correlation between the HP filtered establishment entry rate for new and preexisting firms and the constructed business cycle indicator for the manufacturing sector, using either aggregate manufacturing industrial production or employment.<sup>38</sup> Entry from new firms is more procyclical than from existing firms. Table 16 provides the magnitudes of the cyclical fluctuations in entry by firm age. The boom period is again defined as when the HP filtered cyclical component of the cyclical indicator (either industrial production or employment) is above its trend, and vice versa for the recession period. Then we can calculate the average value of the cyclical component of the entry rate during booms and recessions. Consistent with the correlation pattern from Table 15, establishment entry from new firms shows larger fluctuations across booms and recessions.

One concern about comparing establishment entry from new and existing firms is that

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<sup>35</sup>As measured by variations in sales growth or revenue factor productivity (TFPR) which is not explained by establishment characteristics, economy-wide, or industry-wide factors.

<sup>36</sup>According to BDS terminology, age 0 firms. More specifically, BDS defines age 0 firms in the following way. "Startups are firms with an age of 0. No previous activity is associated with these firms and all its establishments are de novo establishments."

<sup>37</sup>According to BDS terminology, age 1 firms and older.

<sup>38</sup>Refer back to section 2 for details on the construction of these series.

Table 15: Correlation of the entry rate and cyclical indicator by firm age

Firm Age	IP		Employment	
	Correlation	<i>p</i> -value	Correlation	<i>p</i> -value
New	0.4**	0.01	0.44**	0.04
Existing	0.2	0.39	0.27	0.23

Note: *p*-values correspond to whether the correlation is nonzero. “IP” abbreviates industrial production.

Table 16: Magnitudes of fluctuations in the entry rate by firm age (% deviation from trend)

Indicator: Output			
Firm Age	Boom	Recession	<i>p</i> -value
New	1.3%	−1.7%	0.08
Existing	0.6%	−1.2%	0.31

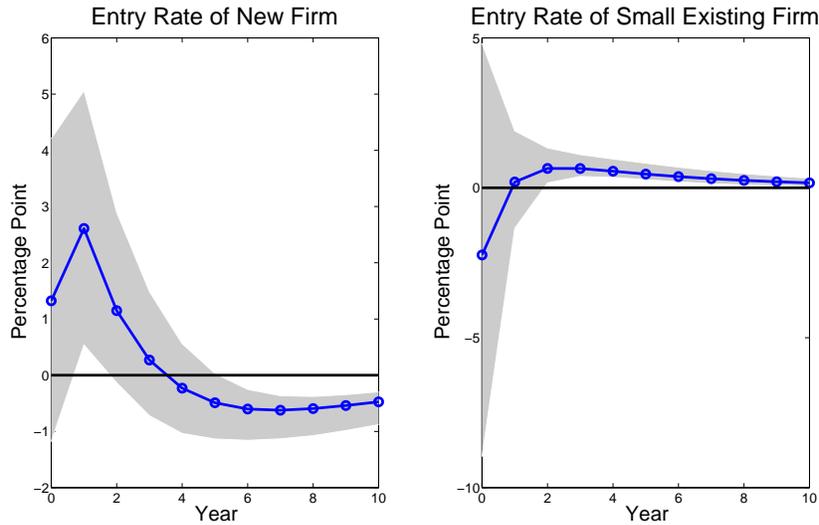
Indicator: Employment			
Firm Age	Boom	Recession	<i>p</i> -value
New	1.4%	−3.4%	0.01
Existing	0.0%	−1.8%	0.34

Note: *p*-values are from the *t*-test of the mean difference in deviations of the entry rate from trend between plant entry from new firms and existing firms.

available information would not be the only difference between them. For example, new firms will likely be more constrained in obtaining external financial sources required to finance entry costs. If financial constraints are relaxed during booms then entry from new firms would be more procyclical than entry from existing firms. In order to deal with this concern, we compare establishment entry from new firms with existing firms whose previous year’s employment total was less than 50. Because it is well known that small firms are more financially constrained, differences in cyclical behavior between new firms and small incumbent firms cannot be solely attributable to cyclical fluctuations in the financial constraint. While this is not perfect - small firms might still have better financial connections than entering firms - it is supportive of our story.

According to the entrepreneurship literature, access to consumer credit and the collateral value of housing are also important financial factors for business startups. Then, additional procyclicality in the entry of new firms might be attributable to cyclical fluctuations in these

Figure 16: Response of entry to the productivity shock by firm age



Note: Shaded region represents 90% bootstrapped confidence interval.

determinants of entry. To deal with this additional concern, we compare entry behavior for new firms and small existing firms using a VAR that adds consumer credit and housing market indicators as exogenous variables in the VAR system<sup>39</sup>.

Recall that using quarterly real output per hour for the manufacturing sector, we previously constructed a re-timed annual productivity measure that is consistent with the BDS's March to March timing. Because of the limited sample size<sup>40</sup>, it is hard - as it was in our previous VAR estimations - to get stable estimates from VARs with 3 endogenous variables (productivity, entry rate from new firms, entry rate from small existing firms) and 2 exogenous variables. Instead, we estimate multiple VARs with 2 endogenous variables and 2 exogenous variables separately, one with (productivity, entry rate from new firms) and another one with (productivity, entry rate from small existing firms). The identification scheme for productivity is that the productivity shock can have a simultaneous effect on the entry rate upon the impact of the shock but a shock to the entry rate can only affect productivity from the next period forward.

Figure 16 shows the response of the establishment entry rate from new firms and small existing firms to the productivity shock. Entry from new firms responds strongly within

<sup>39</sup>Consumer credit (level) and home mortgages (level) from Flow of Funds are used.

<sup>40</sup>BDS is available annually for 1977 to 2012.

the first two years of the shock and then declines sharply. In contrast, small firms respond sluggishly to the shock. This VAR exercise shows that, consistent with the information friction story, establishment entry from new firms is more procyclical than entry from small firms. The different cyclical behavior between new firms and small firms is seemingly unable to be explained by cyclical variations in credit access or housing collateral.

## **6.2. Are plants that entered the market during booms more likely to fail?**

The existence of an imperfect information friction also implies that potential entrants systematically overestimate their expected profit from entry during booms and underestimate their expected profit during recessions. Unfortunately, we are not able to directly observe how potential entrants form these expectations over the business cycle. But, given high entry costs, we can think of exits by one year or two year old establishments as a result of their incorrect estimation of expected profit before entry. If three years of profits cannot cover the cost of entry, then such exits must be a consequence of an incorrect judgment of profits *ex ante*. Therefore, we check if plants that entered the market during a boom period are more likely to exit the market quickly compared to plants that entered the market during recessions.

From the BDS, we categorized establishments that entered between 1980 and 2005<sup>41</sup> into two groups: plants that entered during boom periods<sup>42</sup> are categorized as the ‘boom cohort’ and other plants are categorized as the ‘recession cohort’. For each cohort, the survival rate 1 ~ 3 years after entry is then calculated. Table 17 provides these calculated survival rates. The table illustrates that plants who entered the market during boom periods are more likely to exit the market within three years after entry than cohorts who entered the market during recessions. If early exit actually reflects poor estimation of the expected profit from entry, then these statistically significant differences across boom and recession cohorts indicate that during boom periods potential entrants overestimate expected profit from entry.

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<sup>41</sup>For this exercise we exclude cohorts who were affected by the Great Recession. That is we exclude sample periods after 2006. During the Great Recession, the exit rate for young plants rose. This increased exit rate of young plants during the Great Recession is more likely related to their weak financial structures and economy-wide deteriorations in credit availability than an informational friction.

<sup>42</sup>As before, defined as when HP filtered re-timed annual industrial production of the aggregate manufacturing sector is above trend.

Table 17: Establishment survival rate by cohort

Year After Entry	Boom Cohort	Rec. Cohort	$p$ -value
1	80.8%	81.7%	0.09
2	67.6%	69.0%	0.02
3	57.9%	59.8%	0.00

Note:  $p$ -values are from the  $t$ -test of the mean difference in survival rate between boom and recession cohorts.

If existing firms are actually not subject to the proposed information friction, then for new establishments from existing firms, early exits would not depend on whether the new plant was built during a boom or recession. In general, BDS does not provide statistics based on both firm and establishment age. But in case of the one year old establishments, we can identify how many of them came from new firms or existing firms. We can thus calculate the one year after entry survival rate of the boom and recession cohorts separately for establishments from new firms and from existing firms. Table 18 shows that in case of entry from new firms, cohorts who entered the market during booms are more likely to exit the market within one year. In contrast, for establishments from existing firms, there is no significant difference in their one year survival rate across boom and recession cohorts. This observation indicates that having market experience is critical in making good entry decisions over the business cycle.

Table 18: One year survival rates by cohort and firm age

Firm Age	Boom Cohort	Rec. Cohort	$p$ -value
New Firm	79.6%	80.9%	0.04
Existing Firm	88.0%	87.5%	0.62

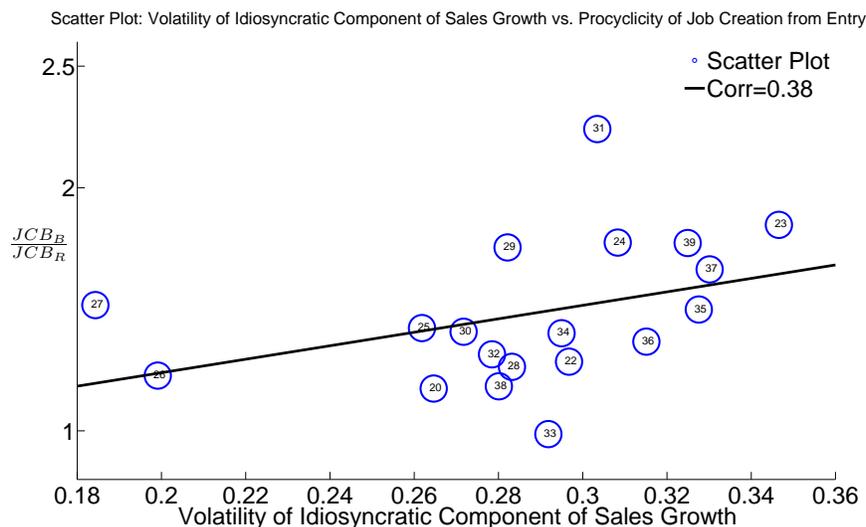
Note:  $p$ -values are from the  $t$ -test of the mean difference in survival rate between boom and recession cohorts.

### 6.3. Is entry more procyclical in sectors where the idiosyncratic shock is more volatile?

Within the manufacturing sector, the volatility of the idiosyncratic component of uncertainty<sup>43</sup> individual plants face varies across disaggregated sectors. According to [Castro et](#)

<sup>43</sup>Uncertainty with respect to any element that might drive profit at the plant level and not the industry level.

Figure 17: Volatility of the idiosyncratic component of sales growth vs. procyclicality of job creation from entry



al. (2015), across 3-digit manufacturing industries, the size of the standard deviation of idiosyncratic shocks to Revenue Total Factor Productivity (TFPR) growth varies from 6.7% (producers of leather soles) to 35.2% (manufactures of non-ferrous metals). For the sectors which have a more volatile plant level idiosyncratic shock, the information friction in separating aggregate and idiosyncratic productivity will be more severe. As a result, sectors with more volatile plant level idiosyncratic shocks should have more procyclical entry.

To test this implication, we need disaggregated data on entry across manufacturing sectors and a measure of the volatility of the plant level idiosyncratic shock. Foster et al. (2006) provides job creation flows from entering plants up to 2-digit manufacturing sectors. Castro et al. (2015) provides the volatility of the idiosyncratic component of plant level sales growth up to 3-digit levels.<sup>44</sup> With the lower-level data on industry shares in Foster et al. (2006), we aggregate up the idiosyncratic volatility to the 2-digit industry code level.

Then, we can use the ratio between the job creation rate from entry during aggregate manufacturing sector boom periods to the creation rate in aggregate manufacturing sector recession periods as a measure of the cyclical entry for each 2-digit sector in terms of entry. Figure 17 shows a scatter plot of the volatility of the idiosyncratic component of sales growth

<sup>44</sup>Both statistics are constructed from the LRD. The volatility calculations are made across a sample period of 1972 to 1997 and job flows through entering plants are available for 1973 to 1998. We therefore used job flow data only for 1973 to 1997.

and the cyclical nature of job creation from entry. The correlation between the two series is 0.38 and the corresponding  $p$ -value against a null of zero is 0.11. Given the small number of observations, the potential for a very significant correlation is quite limited, but there is still a noticeably positive relation.

## 7. Additional Issues

In this section, we briefly touch on a few miscellaneous issues and robustness checks. We want to check whether the failure of the standard full information model in explaining entry and exit results from a too stringent calibration of the Frisch elasticity of labor supply. We also want to check whether our baseline model economy with the information friction can jointly explain the cyclical nature of entry and exit together with the cyclical nature of continuing plant job flows. Another issue we are interested in is whether we can generate cyclical nature in entry and exit that is consistent with the data by adding a countercyclical entry cost to the full information economy. Finally, we try to refine the expected profit potential entrants can obtain on entry by utilizing information from additional aggregate variables in their entry decision.

### 7.1. High Frisch Elasticity of Labor Supply

One concern with investigating the cyclical behavior of entry and exit with a standard macro value for the Frisch elasticity of labor supply in this context is that the model economy's aggregate employment is less volatile than actual manufacturing sector aggregate employment. We want to check whether the full information model can match the cyclical nature of entry and exit once we use a higher Frisch elasticity, which would yield model implied aggregate employment dynamics that are closer to the observed aggregate employment dynamics in the manufacturing sector.

Table 19 shows the simulated cyclical nature of wages and aggregate employment depending on the Frisch elasticity of labor supply. Due to an income effect, even with a Frisch elasticity of 15, aggregate employment is less volatile than in the data. But, this remains informative of the effect labor supply elasticity can have on entry and exit.

As the Frisch elasticity increases, aggregate consumption grows more quickly, which

Table 19: Cyclical properties of wages and employment under different Frisch elasticities

	Wage		Employment	
	S.D.(% to $y$ )	AR(1)	S.D.(% to $y$ )	AR(1)
Data	35	0.59	80	0.61
Full Info. 1.5	61	0.52	34	0.45
Full Info. 15	45	0.6	49	0.51

Note: In the data, wages and employment are the BLS provided manufacturing sector real compensation per hour and employment series.

means that wages grow quickly as well. Additionally, in the high elasticity economy, on impact of the shock, the conditional mean of the stochastic discount factor drops by more compared to in the baseline model, which means that future profit is more aggressively discounted. In the full information environment, potential entrants can properly forecast wage and consumption dynamics. Here, the one period lag required for potential entrants to actually start operating is somewhat critical. Because of this one period lag, potential entrants cannot benefit from the dampened response of equilibrium wages. So, within a range for the Frisch elasticity of 1.5 to 15, the effect of the increase in the Frisch elasticity of labor supply comes through amplification of the cyclicalities of continuing plants' job flows. This exercise shows that the full information model's failure in matching the cyclicalities of entry and exit cannot be fixed simply by using a high Frisch elasticity of labor supply.

Table 20: Entry, exit and job flows with different Frisch elasticities

	Entry		Exit	
	Boom(%)	Recession(%)	Boom(%)	Recession(%)
Data	8.1	3.4	5.8	5.1
Full Info. 1.5	6.3	5.9	6.0	6.2
Full Info. 15	6.3	5.9	6.0	6.2

	Job Creation		Job Destruction	
	Boom(%)	Recession(%)	Boom(%)	Recession(%)
Data	7.9	6.9	6.8	8.5
Full Info. 1.5	8.7	8.1	8.7	9.1
Full Info. 15	8.5	7.6	8.2	9.1

Note: Entry and exit data comes from [Lee and Mukoyama \(2015a\)](#). Job creation and destruction flows (of continuing plants) comes from [Foster et al. \(2006\)](#). Booms and recessions are categorized depending on whether the output growth rate of a given period is above or below the average output growth rate.

Figure 18: Impulse response of entry and wage forecasts

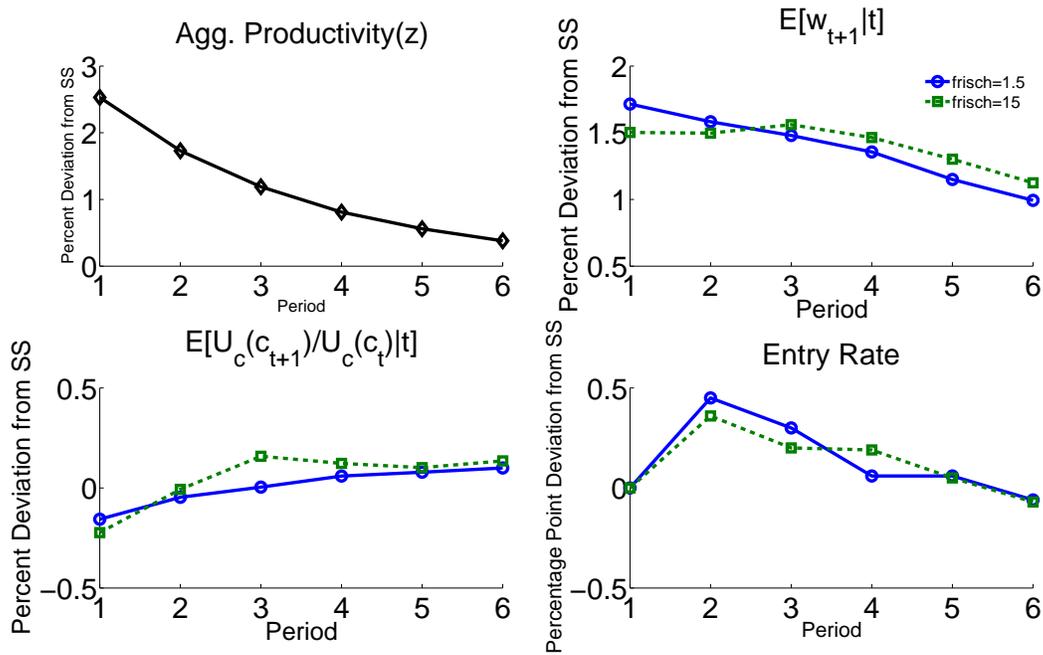
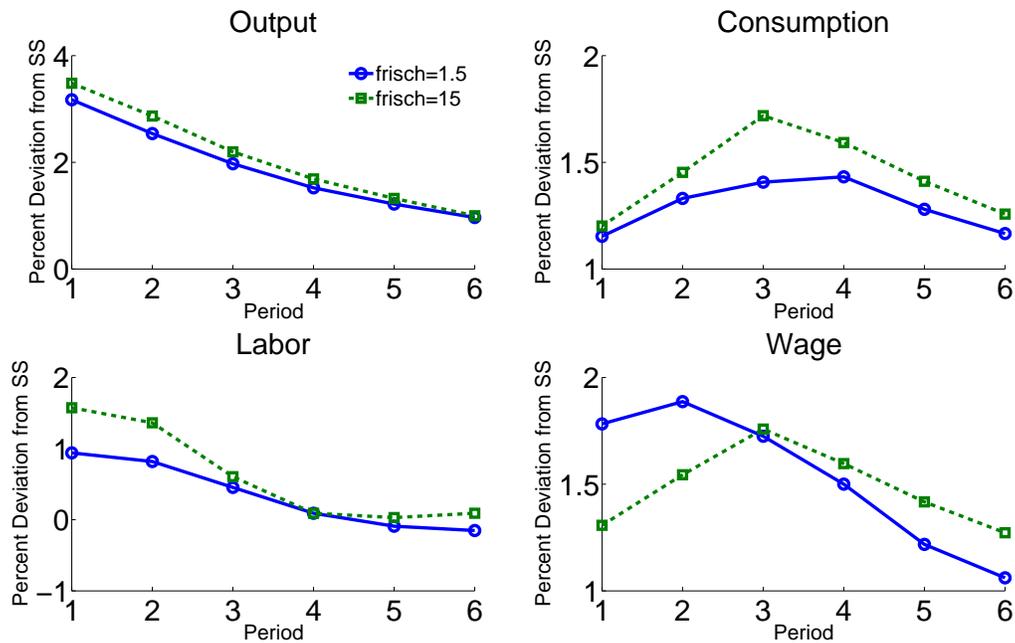


Figure 19: Impulse response of aggregate variables



## 7.2. Continuing Plant Job Flows in the Baseline Economy

One of the problems with the baseline economy is that because the entry margin responds too strongly and labor demand is forward-looking and hence sluggish, the net job creation

by incumbent plants is not procyclical. This is at odds with the data. Table 21<sup>45</sup> shows the baseline model implies almost acyclical job flows from continuing plants.

Table 21: Job creation and destruction rates

	Data		Baseline	
	Boom(%)	Recession(%)	Boom(%)	Recession(%)
JC from Entry	1.8	1.2	4.3	2.5
JC from Continue	7.9	6.9	8.4	8.4
JD from Exit	2.6	2.3	2.8	2.7
JD from Continue	6.8	8.5	8.8	8.9

Note: Job flows data is from [Foster et al. \(2006\)](#).

Three things in our baseline calibration strategy are key for the low cyclicity of continuing plant job flows. We tackle them separately.

First, as already mentioned in Section 4, the job creation rate from entering plants implied by statistics provided from [Lee and Mukoyama \(2015a\)](#) is higher than that provided from [Foster et al. \(2006\)](#) job flows data. That is why we have targeted slightly smaller average employment sizes of entering plants than the corresponding statistics from [Lee and Mukoyama \(2015a\)](#) and higher total job flows rates than the corresponding statistics from [Foster et al. \(2006\)](#). But still our baseline calibration strategy overestimates job creation from entering plants compared to the statistics from [Foster et al. \(2006\)](#). In our baseline calibration,  $\frac{\text{JC from Entry}}{\text{Total JC}} = 30\%$ , while [Foster et al. \(2006\)](#) finds it as 17%. This means that compared to the job flows data, our baseline calibration assigns too large an employment share to entering plants.

Second, we chose a stochastic process for aggregate productivity process such that the cyclical behavior of model aggregate output matches the cyclical behavior of net manufacturing sector real output that is explained by productivity shocks. As a result, the baseline economy's standard deviation of aggregate output is only 84% of the standard deviation of the cyclical component of the actual manufacturing sector's output. In the baseline model, less volatile aggregate productivity does not have a uniform effect across potential entrants

<sup>45</sup>Booms and recessions are defined again by whether manufacturing sector output growth is above or below average. [Foster et al. \(2006\)](#)'s job flows data are constructed using the LRD over 1992 ~ 1997. The job creation rate from entering plants is defined as the number of jobs created from entering plants as a percentage of total manufacturing sector employment in a given period. Other job flows are similarly defined.

and incumbent plants. As the volatility of the aggregate shock shrinks, the imperfect information problem of potential entrants becomes severe. Therefore, entry volatility does not monotonically decrease with aggregate productivity volatility. In contrast, because incumbent plants can directly observe aggregate productivity, incumbent job flow volatility monotonically decreases with aggregate productivity volatility. Therefore, specifying the aggregate productivity process to only account for a fraction of the actual manufacturing sector’s cyclical variation reduces job flows among continuing plants.

Third, as we discussed in Section 7.1, with a standard macro value for the Frisch elasticity of labor supply, we cannot match volatility of aggregate employment in the manufacturing sector. Further, job flows from continuing plants are more sensitive to the magnitude of the Frisch elasticity.

The question is then whether there exists a reasonable alternative calibration strategy that can jointly explain the cyclical behavior of entry and exit rate together with procyclical net job creation from continuing plants. We will try two things.

One alternative calibration strategy is to set a Frisch elasticity of labor supply of 2 (still within the range of standard macro Frisch elasticities) and target a smaller employment contribution from entering plants:  $\frac{\text{JC from Entry}}{\text{Total JC}} = 24\%$ . Additionally, the persistence and volatility of the aggregate shock is adjusted such that the model implied volatility of output matches actual (not just the portion explained by variations in TFP) manufacturing sector output. This requires aggregate shock persistence to increase from 0.68 to 0.73 and the standard deviation of the innovation to also rise from 2.5% to 3%.<sup>46</sup>

A second alternative is to use a higher Frisch elasticity of labor supply - 15 - with the same aggregate productivity process and matching exactly the same targets as in the baseline calibration strategy. Table 22 summarizes the fluctuations in entry, exit, and job flows within the baseline model under these alternative calibrations.<sup>47</sup> Either alternative can match the data cyclicity of entry and exit and simultaneously generate procyclical net job creation from continuing plants.

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<sup>46</sup>As per our main calibration, the idiosyncratic productivity process is calibrated with equal persistence, so it increases as well. All other moments are matched with the same targets used in the baseline calibration strategy.

<sup>47</sup>We labeled the first alternative as ‘smaller entry size’ and the second as ‘high Frisch elasticity’.

Table 22: Cyclicity of entry, exit and job flows in alternative calibration strategies

	Data		Smaller Entry Size	
	Boom(%)	Recession(%)	Boom(%)	Recession(%)
Entry Rate	8.1	3.4	7.7	4.2
Exit Rate	5.8	5.1	5.7	5.7
JC from Entry	1.8	1.2	3.6	2.0
JC from Continue	7.9	6.9	9.8	9.5
JD from Exit	2.6	2.3	2.7	2.7
JD from Continue	6.8	8.5	9.4	9.8

	Data		High Frisch Elasticity	
	Boom(%)	Recession(%)	Boom(%)	Recession(%)
Entry Rate	8.1	3.4	7.7	4.3
Exit Rate	5.8	5.1	5.8	5.8
JC from Entry	1.8	1.2	4.3	2.5
JC from Continue	7.9	6.9	8.3	7.9
JD from Exit	2.6	2.3	2.8	2.7
JD from Continue	6.8	8.5	8.1	8.9

### 7.3. Countercyclical Entry Costs

One of the mechanisms considered in the previous literature<sup>48</sup> to jointly explain strongly procyclical entry and acyclical exit is a countercyclical entry cost within a full information rational expectations structure. We therefore test whether we can generate realistic cyclicity in entry and exit by adding a countercyclical entry cost to the full information model economy. We modify our full information model economy so that if aggregate productivity in a given period is  $\exp(z)$  then the entry cost is  $\frac{c_e}{\exp(z)^{1.5}}$ . That is, if aggregate productivity increases 1% then the entry cost becomes cheaper by 1.5%.

One interesting observation from the countercyclical entry cost model is that magnitude of fluctuations in entry critically hinges on how the boom and recession periods are classified. In Table 23<sup>49</sup>, we present fluctuations in entry and exit when we classify booms and recessions as depending on whether aggregate output growth is above or below the average growth rate. According to this classification, the countercyclical entry cost does not generate enough

<sup>48</sup>Especially, [Lee and Mukoyama \(2015b\)](#) suggested this mechanism.

<sup>49</sup>We labeled the model economy with a countercyclical entry cost as ‘Cyclical Cost’.

amplification in entry.

In Table 24, we provide the magnitude of the fluctuations in entry when we classify booms and recessions based on the level of aggregate productivity. Here, a given period is classified as a boom if  $z_t > 0$  and vice versa. In the case of the baseline model entry fluctuations do not vary depending on these classification differences, but with countercyclical entry costs we find stronger amplification in entry under this alternate classification scheme.

Table 23: Magnitude of fluctuations when the cycle is based on output growth

	Entry Rate(%)		Exit Rate(%)	
	Boom	Rec.	Boom	Rec.
Data	8.1	3.4	5.8	5.1
Baseline	7.6	4.5	5.8	5.8
Full Info	6.3	5.9	6.0	6.2
Cyclical Cost	6.5	5.7	6.0	6.2

Table 24: Magnitude of fluctuations when the cycle is based on productivity levels

	Boom	Rec.
Baseline	7.8	4.5
Full Info	6.6	5.6
Cyclical Cost	7.2	5.1

To get more insight about this result, we plot model generated business cycle episodes. In Figure 20, we plot fluctuations in the entry rate together with the level of aggregate productivity and in Figure 21, we plot fluctuations in the entry rate together with output growth.<sup>50</sup> In case of the countercyclical entry cost model, because potential entrants can properly forecast the effects of the amplified entry responses on future factor prices, amplification of entry at business cycle turning points is weaker than in the baseline model. But given the persistence of aggregate productivity, entry is also persistently cheaper during booms and vice versa during recessions. As a result, in the countercyclical entry cost model, entry stays high during boom periods. Conversely, in the baseline economy the response of

<sup>50</sup>The output growth rate time series from both economies are not identical but show very similar dynamics. So we just plotted the output growth rate from the baseline economy.

Figure 20: Aggregate productivity and entry

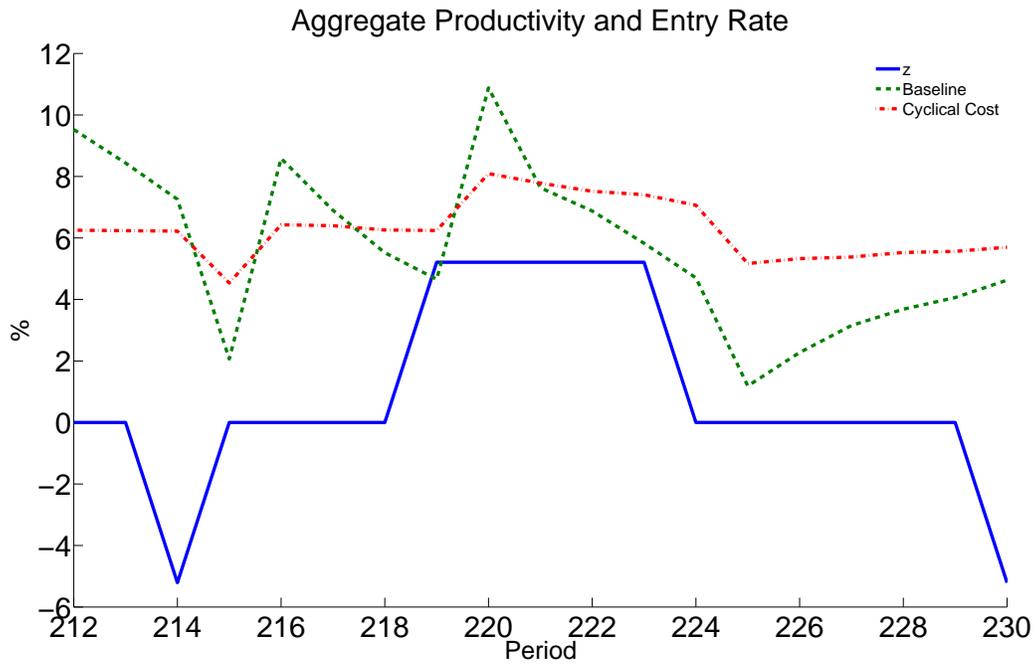
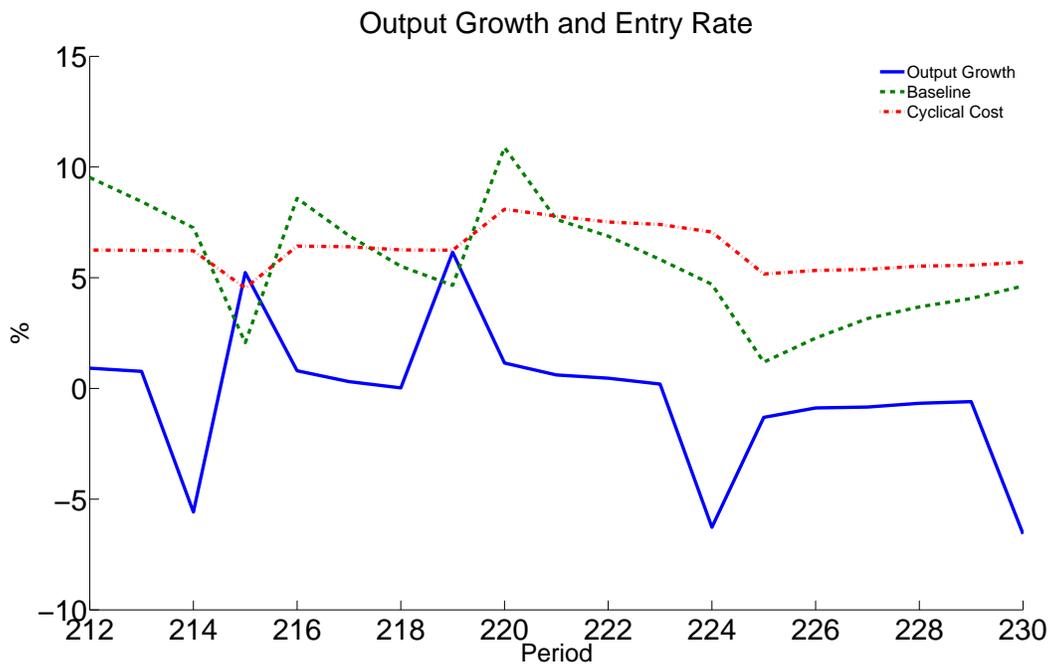


Figure 21: Output growth and entry



entry is strongly amplified at business cycle turning points and rapidly drops as potential entrants learn more about the aggregate state through the dynamics of the equilibrium wage.

The output growth rate captures turning points in the business cycle. To generate huge

swings in the entry rate along the business cycle classified by output growth (as [Lee and Mukoyama \(2015a\)](#) classified booms and recessions), we need very specific dynamics for entry: high at the peak and then declining quickly. [Figure 21](#) shows this point clearly. We can see that variations in the entry rate in the baseline economy are synchronized with variations in the output growth rate. In contrast, in the countercyclical entry cost model, variations in the entry rate are too persistent and the magnitude of the amplification at the peak of the business cycle is not large enough.

Additionally, in the countercyclical entry cost model, because potential entrants can properly forecast the effect of the mass entry on future factor prices, the entry response cannot become strong enough for the exit rate to become acyclical. The correlation between the HP filtered exit rate and output in the countercyclical entry cost model is still  $-0.49$ .

To sum up, the countercyclical entry cost model with full information cannot generate enough amplification in entry if we classify booms and recessions based on the output growth rate, nor can it generate an acyclical exit rate.

#### 7.4. Information from Higher Order Moments

In the baseline model economy, potential entrants get information about the aggregate status from an exogenous signal ( $z_t + q_{t,i}$ ), the current equilibrium wage ( $w_t$ ), and the aggregate capital stock ( $K_t$ ). Given that potential entrants cannot fully recover the current aggregate state by observing these variables, it is possible for potential entrants to learn more about aggregate productivity by observing more aggregate variables. If so, by how much can they improve their estimates, and how does that translate into improved profits? Even though it is not explicitly modeled here, if getting more information about the aggregate state costs more than the additional expected profit potential entrants expect from utilizing the additional information in their entry decision, then they would not use that specific variable in their learning problem.

Quantifying the value of more information is thus necessary to evaluate whether the baseline model potential entrants should actually be using a larger information set. To be precise, we want to answer following question: Given that all incumbent plants and potential entrants make their decision as in the baseline model, if one potential entrant was to deviate from this equilibrium and use an additional aggregate variable in their learning problem,

how much additional profit could they expect?

Consider the situation where a deviating potential entrant uses the second moment of the incumbent plant distribution over capital as an additional variable. This means that in addition to (13) and (14), the deviating potential entrant is aware of a projection equation for the second moment of the capital distribution within the baseline economy:

$$\log SK = \beta_{SK,c} + \beta_{SK,z}z + \epsilon_{SK}, \quad Var(\epsilon_{SK}) = \sigma_{SK}^2 \quad (38)$$

With this additional piece of information, the potential entrant's Kalman filter measurement equation is now:

$$\underbrace{\begin{bmatrix} a_t \\ \log w_t \\ \log K_t \\ \log SK_t \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} 1 & 1 \\ \beta_{w,z} & 0 \\ \beta_{K,z} & 0 \\ \beta_{SK,z} & 0 \end{bmatrix}}_{\beta} \begin{bmatrix} z_t \\ q_t \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \beta_{w,c} \\ \beta_{K,c} \\ \beta_{SK,c} \end{bmatrix}}_{\beta_c} + \begin{bmatrix} 0 \\ \epsilon_w \\ \epsilon_K \\ \epsilon_{SK} \end{bmatrix} \quad (39)$$

Other than this extended measurement equation, the deviating potential entrant has an otherwise unchanged problem compared to the baseline model. Because the deviator has a different information set than other potential entrants, just comparing ex-ante expected profit from entry between them is meaningless. Rather, comparing the average (out of many different idiosyncratic shock sequences) realized profit from deviating is a better measure of whether the deviation is worthwhile.

We generate two sets of unbalanced panel data comprised of roughly 60,000 plants over 100 years of business cycle fluctuations. Both panels are calculated from the baseline model. In the 'deviating' panel, potential entrants make entry decisions using information from  $(z_t + q_{t,i}, w_t, K_t, SK_t)$  and in the 'usual' panel set, potential entrants make entry decisions using information from  $(z_t + q_{t,i}, w_t, K_t)$ . Then, for all the plants who have ever operated, it is possible to calculate cash flows generated from each plant taking into account entry costs and exit values. For example, consider a specific potential entrant who decides to enter in period 1 and operated for periods 2 ~ 10. Then, the cash flows generated from this specific plant are given as follows.

$$-u'(C_1)c_e + \sum_{t=2}^{t=10} u'(C_t) [y_t - w_t n_t - AC_t^k - AC_t^n] + u'(C_{10})p_s(1 - \delta)k_{10}$$

Note that cash flows in different periods are denominated using each period’s representative household marginal utility of consumption.

In both data sets, we calculate value of this cash flow for every single plant who ever operated. According to this calculation, the ‘deviating’ panel has 0.2% higher cash flows on average. That is to say, if cost of getting extra information about aggregate productivity using the second moment of the capital distribution is higher than 0.2% of average ex-post profits of operating plants in the baseline economy, then potential entrants could not profitably use this extra information. Given the difficulty of measuring the costs of getting information from aggregate observables, however, this 0.2% number only conveys very limited information.

This exercise might provide a different view point on the [Krusell and Smith \(1998\)](#) environment. When the quasi-aggregation result does not hold and there are a non-zero measure of agents who can improve their price forecasting by using higher order moments, how much incentive do these agents have to deviate to a more sophisticated boundedly-rational behavior? If the incentive to deviate is small enough, then even though the quasi-aggregation result does not hold for all agents in the economy<sup>51</sup>, aggregate dynamics derived from individual agents’ boundedly-rational behavior can still characterize a valid approximate equilibrium.

## 8. Concluding Remarks

Using the general equilibrium heterogeneous plant model where the growth of plants is governed by non-convex capital and labor adjustment costs, we showed that once general equilibrium forces are disciplined by standard values for risk aversion and Frisch elasticity, a standard industry dynamics model cannot generate empirically realistic cyclical behavior in entry and exit. This paper provides a mechanism that can jointly match the observed cyclicity of entry and exit with state-invariant entry and operating costs: an information friction among potential entrants that are considering entry. Specifically, potential entrants are unable to disentangle aggregate productivity from their own idiosyncratic productivity when considering entry. In response to a positive aggregate shock, they partially misinterpret the aggregate boom as being unique to them and enter too quickly. This not only amplifies

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<sup>51</sup>In other words, not every agent in the economy has precise price forecasting abilities.

the response of entry, through general equilibrium effects it renders exit acyclical, as seen in the data.

This baseline model with incomplete information can also explain the cyclical behavior of job flows across establishments of different ages. Notably, we observe that job creation is much more volatile than job destruction among young plants. This paper shows that even with strictly disciplined general equilibrium forces, by amplifying the entry response through the imperfect information of potential entrants, the introduction of plant dynamics can work as a quantitatively important internal propagation mechanism.

Even though we cannot directly test for the presence of an information friction, several testable implications of the model economy are consistent with the data: plant entry from new firms is more procyclical than entry from existing firms after controlling for cyclical variations in credit availability; plants from new firms that entered the market during booms are more likely to exit the market rapidly compared to those that entered the market during recessions; industries where the idiosyncratic component of productivity is more important exhibit more procyclical job creation from entry over the business cycle.

One limitation of our framework is that the endogenous variables that potential entrants can use as source of learning about the aggregate state are exogenously fixed. Introducing an endogenous choice of observables through a rational inattention style argument could be an interesting extension of the developed framework.

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## A. Derivation of Stationary Kalman Gain

### Unobservable Stochastic Process

$$z_{t+1} = \rho_z z_t + \epsilon_{z,t+1} \quad (40)$$

$$q_{t+1} = (1 - \rho_q)\bar{q} + \rho_q q_t + \epsilon_{q,t+1} \quad (41)$$

### Observable Variables

$$a_t = z_t + q_t \quad (42)$$

$$\log w_t = \beta_{w,c} + \beta_w z_t + \epsilon_w \quad (43)$$

$$\log K_t = \beta_{K,c} + \beta_K z_t + \epsilon_K \quad (44)$$

### Measurement Equation

$$\underbrace{\begin{bmatrix} a_t \\ \log w_t \\ \log K_t \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} 1 & 1 \\ \beta_w & 0 \\ \beta_K & 0 \end{bmatrix}}_{\beta} \underbrace{\begin{bmatrix} z_t \\ q_t \end{bmatrix}}_{\beta_c} + \underbrace{\begin{bmatrix} 0 \\ \beta_{w,c} \\ \beta_{K,c} \end{bmatrix}}_{\beta_c} + \underbrace{\begin{bmatrix} 0 \\ \epsilon_w \\ \epsilon_K \end{bmatrix}}_{R_t} \quad (45)$$

$$\text{where } \begin{pmatrix} \epsilon_w \\ \epsilon_K \end{pmatrix} \sim i.i.d. \text{ N} \left[ 0, \underbrace{\begin{pmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_K^2 \end{pmatrix}}_{R_t} \right]$$

### Transition Equation

$$\begin{bmatrix} z_t \\ q_t \end{bmatrix} = \underbrace{\begin{bmatrix} \rho_z & 0 \\ 0 & \rho_q \end{bmatrix}}_T \begin{bmatrix} z_{t-1} \\ q_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ (1 - \rho_q)\bar{q} \end{bmatrix}}_{T_c} + \underbrace{\begin{bmatrix} \epsilon_z \\ \epsilon_q \end{bmatrix}}_{Q_t} \quad (46)$$

$$\text{where } \begin{pmatrix} \epsilon_z \\ \epsilon_q \end{pmatrix} \sim i.i.d. \text{ N} \left[ 0, \underbrace{\begin{pmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_q^2 \end{pmatrix}}_{Q_t} \right]$$

### Notation

Denote the prior mean and variance of state variables before realization of time  $t$  observations as  $\mu_{t|t-1}$  and  $V_{t|t-1}$  respectively.

$$\mu_{t|t-1} = \begin{bmatrix} \mu_z \\ \mu_q \end{bmatrix}, \quad V_{t|t-1} = \begin{bmatrix} \sigma_z^2 & \sigma_{zq} \\ \sigma_{qz} & \sigma_q^2 \end{bmatrix}$$

## Updating Equation

$$\mu_{t|t} = \mu_{t|t-1} + \underbrace{V_{t|t-1}\beta_t'H_t^{-1}}_{\text{Kalman Gain} \equiv G_t} (Y_t - \beta_t\mu_{t|t-1} - \beta_c) \quad (47)$$

$$V_{t|t} = V_{t|t-1} - V_{t|t-1}\beta_t'H_t^{-1}\beta_tV_{t|t-1} \quad (48)$$

$$\text{where } H_t \equiv \beta_tV_{t|t-1}\beta_t' + R_t$$

We can then do some rewriting:

$$\begin{aligned} H_t &= \begin{bmatrix} 1 & 1 \\ \beta_w & 0 \\ \beta_K & 0 \end{bmatrix} \begin{bmatrix} \sigma_z^2 & \sigma_{zq} \\ \sigma_{qz} & \sigma_q^2 \end{bmatrix} \begin{bmatrix} 1 & \beta_w & \beta_K \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_w^2 & 0 \\ 0 & 0 & \sigma_K^2 \end{bmatrix} \\ &= \begin{bmatrix} \sigma_z^2 + 2\sigma_{zq} + \sigma_q^2 & \beta_w(\sigma_z^2 + \sigma_{zq}) & \beta_K(\sigma_z^2 + \sigma_{zq}) \\ \beta_w(\sigma_z^2 + \sigma_{zq}) & \beta_w^2\sigma_z^2 + \sigma_w^2 & \beta_w\beta_K\sigma_z^2 \\ \beta_K(\sigma_z^2 + \sigma_{zq}) & \beta_w\beta_K\sigma_z^2 & \beta_K^2\sigma_z^2 + \sigma_K^2 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} \mu_z \\ \mu_q \end{bmatrix}_{t|t} = \begin{bmatrix} \mu_z \\ \mu_q \end{bmatrix}_{t|t-1} + \begin{bmatrix} \sigma_z^2 + \sigma_{zq} & \beta_w\sigma_z^2 & \beta_K\sigma_z^2 \\ \sigma_{qz} + \sigma_q^2 & \beta_w\sigma_{qz} & \beta_K\sigma_{qz} \end{bmatrix} \times H_t^{-1} \times \begin{bmatrix} a_t - (\mu_z + \mu_x)_{t|t-1} \\ \log w_t - \beta_w\mu_z|_{t|t-1} - \beta_{w,c} \\ \log K_t - \beta_K\mu_z|_{t|t-1} - \beta_{K,c} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_z^2 & \sigma_{zq} \\ \sigma_{qz} & \sigma_q^2 \end{bmatrix}_{t|t} = \begin{bmatrix} \sigma_z^2 & \sigma_{zq} \\ \sigma_{qz} & \sigma_q^2 \end{bmatrix}_{t|t-1} - \begin{bmatrix} \sigma_z^2 + \sigma_{zq} & \beta_p\sigma_z^2 & \beta_k\sigma_z^2 \\ \sigma_{qz} + \sigma_q^2 & \beta_p\sigma_{qz} & \beta_k\sigma_{qz} \end{bmatrix} \times H_t^{-1} \times \begin{bmatrix} \sigma_z^2 + \sigma_{zq} & \sigma_{qz} + \sigma_x^2 \\ \beta_p\sigma_z^2 & \beta_p\sigma_{qz} \\ \beta_k\sigma_z^2 & \beta_k\sigma_{qz} \end{bmatrix}$$

## Prediction Equation

$$\begin{bmatrix} \mu_z \\ \mu_q \end{bmatrix}_{t+1|t} = \begin{bmatrix} \rho_z\mu_z \\ (1 - \rho_q)\bar{q} + \rho_q\mu_q \end{bmatrix}_{t|t} \quad (49)$$

$$\begin{bmatrix} \sigma_z^2 & \sigma_{zq} \\ \sigma_{qz} & \sigma_q^2 \end{bmatrix}_{t+1|t} = \begin{bmatrix} \rho_z^2\sigma_z^2 + \sigma_{\epsilon,z}^2 & \sigma_{zq} \\ \sigma_{qz} & \rho_x^2\sigma_x^2 + \sigma_{\epsilon,q}^2 \end{bmatrix}_{t|t} \quad (50)$$

Iterations on (48) and (50) with a positive semi-definite  $V_{0|0}$  converge to the stationary Kalman gain ( $G$ ) and prediction covariance ( $V$ ).<sup>52</sup> These converged stationary Kalman gain and prediction variance are used in the potential entrants' learning problem.

The measurement equations associated with wages (43) and the aggregate capital stock (44) are endogenously determined via stochastic simulation of the baseline model and are given as follows:

$$\ln w_t = 0.65 + 1.03z_t, \quad \sigma_{\epsilon_w} = 0.028 \quad (51)$$

$$\ln K_t = 0.93 + 0.53z_t, \quad \sigma_{\epsilon_K} = 0.042 \quad (52)$$

<sup>52</sup>Detailed conditions that guarantee convergence are provided in [Anderson et al. \(1996\)](#).

## B. Computation

### B.1. Solution Algorithm

Our solution algorithm is based on [Krusell and Smith \(1998\)](#) and [Khan and Thomas \(2008\)](#). We need two main modifications compared to [Khan and Thomas \(2008\)](#). At first, because of the finite Frisch elasticity of labor supply, we need a forecasting rule for both the marginal utility of consumption and wages. Second, in addition to the forecasting rules for prices and aggregate capital stock, we need consistent projection equations (see, (13) and (14) in Section 3.2) for wages and aggregate capital stock onto aggregate productivity. This enables potential entrants to learn about aggregate productivity by observing current period equilibrium wages and the aggregate capital stock.

1. Approximate the distribution of plants by the aggregate capital stock. Make initial guesses for (i) the log-linear law of motion for the aggregate capital stock, (ii) the forecasting rule for wages, (iii) the forecasting rule for marginal utility of consumption, (iv) the log-linear projection equation of wages onto aggregate productivity, and (v) the log-linear projection equation of aggregate capital stock onto aggregate productivity.

$$(i) \ln K_{t+1} = \kappa_0^0 + \kappa_1^0 \ln K_t + \kappa_2^0 z_t \quad (53)$$

$$(ii) \ln w_t = a_0^0 + a_1^0 \ln K_t + a_2^0 z_t \quad (54)$$

$$(iii) \ln p_t = b_0^0 + b_1^0 \ln K_t + b_2^0 z_t \quad (55)$$

$$(iv) \log w_t = \beta_{w,c}^0 + \beta_{w,z}^0 z + \epsilon_w, \quad S.D.(\epsilon_w) = \sigma_w^0 \quad (56)$$

$$(v) \log K_t = \beta_{K,c}^0 + \beta_{K,z}^0 z + \epsilon_K, \quad S.D.(\epsilon_K) = \sigma_K^0 \quad (57)$$

$p_t$  in (55) represents the marginal utility of consumption  $C_t^{-\sigma_c}$ . The superscript for each coefficient represents an index for the iteration. Note that (56) and (57) are used as the measurement equation in potential entrants' Kalman filter problem. Unlike the forecasting rules, potential entrants' perceived variance of the residual does matter. The goal of the whole algorithm is then to find a set of

$$\{(\kappa_0, \kappa_1, \kappa_2), (a_0, a_1, a_2), (b_0, b_1, b_2), (\beta_{w,c}, \beta_{w,z}, \sigma_w), (\beta_{K,c}, \beta_{K,z}, \sigma_K)\}$$

such that they are consistent with the actual aggregate dynamics of the model economy.

2. To make sure both the goods and labor market are actually cleared in each period we follow a two step procedure suggested by [Krusell and Smith \(1998\)](#). One, current period market clearing consumption and wages come from explicit market clearing conditions. Two, forecasting rules given by (53)~(55) are only used when calculating incumbent and potential entrant perceptions of future prices. Following [Khan and Thomas \(2008\)](#), we will describe the optimization problems of incumbents and potential entrants in terms of the marginal-utility transformed Bellman equation. That is, we will denominate plants' profit and costs by the marginal utility of consumption. We will denote the actual market clearing wage and marginal utility of consumption as  $w$  and  $p$ , and the wage and marginal utility of consumption implied by the forecasting rules, (54) and (55), as  $W(z, K)$  and  $P(z, K)$ .

Step.1: Value function based on forecasted prices

Incumbent plants' value functions are defined on 5 dimensions of state variables. 3 of them are individual state variables (last period's labor ( $n_{-1}$ ), individual capital stock ( $k$ ), idiosyncratic productivity ( $x$ )) and 2 of them are aggregate state variables (aggregate productivity ( $z$ ), aggregate capital stock ( $K$ )). For state variables that take a continuous value ( $n_{-1}$ ,  $k$ ,  $K$ ), we used trilinear interpolation when we needed to evaluate off-grid-point values. The AR(1) processes specified for an incumbents' idiosyncratic shock and aggregate productivity are discretized using the [Tauchen \(1986\)](#) method.

Denote this first stage value function as  $\hat{V}$ . For each of the aggregate state variables ( $z, K$ ), we can calculate the wage and marginal utility of consumption implied by a combination of aggregate state variables using the price forecasting rules given by (54) and (55).

$$W(z, K) = a_0^0 + a_1^0 \ln K + a_2^0 z \quad (58)$$

$$P(z, K) = b_0^0 + b_1^0 \ln K_t + b_2^0 z \quad (59)$$

For each point in the state space, ( $n_{-1}, k, x; z, K$ ), we can re-write incumbent plants' value function ((6)~(8) given in Section 3.1) combined with (58) and (59).

$$\hat{V}(n_{-1}, k, x; z, K) = \int \max\{\hat{V}_c(n_{-1}, k, x; z, K) - \xi, \hat{V}_x(n_{-1}, k, x; z, K)\} dG(\xi) \quad (60)$$

$$\begin{aligned}\hat{V}_c(n_{-1}, k, x; z, K) &= \max_{\{i, n\}} P(z, K)(y - W(z, K)n - i - AC^k(k, i) - AC^n(n_{-1}, n)) \\ &\quad + \beta \mathbb{E}[\hat{V}(n, k', x'; z', K')] \end{aligned} \quad (61)$$

$$\hat{V}_x(n_{-1}, k, x; z, K) = \max_{\{n\}} P(z, K)(y - W(z, K)n - AC^n(n_{-1}, n) - AC^n(n, 0) + p_s(1 - \delta)k) \quad (62)$$

$$\ln K' = \kappa_0^0 + \kappa_1^0 \ln K + \kappa_2^0 z \quad (63)$$

Step.2: Search for actual market clearing ( $w, p$ )

The converged value function ( $\hat{V}(n_{-1}, k, x; z, K)$ ) from step 1 will be used to evaluate forward values in the incumbent and potential entrant optimization problems inside the actual stochastic simulation. At each point in time, incumbents solve the following problem:

$$V(n_{-1}, k, x; z, K) = \int \max\{V_c(n_{-1}, k, x; z, K) - \xi, V_x(n_{-1}, k, x; z, K)\} dG(\xi) \quad (64)$$

$$\begin{aligned}V_c(n_{-1}, k, x; z, K) &= \max_{\{i, n\}} p(y - wn - i - AC^k(k, i) - AC^n(n_{-1}, n)) \\ &\quad + \beta \mathbb{E}[\hat{V}(n, k', x'; z', K')] \end{aligned} \quad (65)$$

$$V_x(n_{-1}, k, x; z, K) = \max_{\{n\}} p(y - wn - AC^n(n_{-1}, n) - AC^n(n, 0) + p_s(1 - \delta)k) \quad (66)$$

$$\ln K' = \kappa_0^0 + \kappa_1^0 \ln K + \kappa_2^0 z \quad (67)$$

The main difference compared to the step 1 optimization problem is here the optimization problem depends on actual prices ( $p, w$ ) instead of forecasting rule implied prices ( $P(z, K), W(z, K)$ ).

Now think about potential entrants' entry decision. Because potential entrants learn about the current aggregate productivity from the current actual market clearing wage, in the process of searching for the current period market clearing ( $p, w$ ), whenever a different value of  $w$  is proposed, potential entrants' Kalman filter problem should be re-solved and the expected value from entry should be re-evaluated. In each period, for a given predetermined beginning of period aggregate capital stock and proposed wage,  $w$ , potential entrants solve their Kalman filter problem composed of (15)~(22) in Section 3.2.

$$V_{en}(a, \mu_{t|t-1}^z, \mu_{t|t-1}^q; w, K) = -pk'_{en} + \beta \mathbb{E}[\hat{V}(0, k'_{en}, x'; z', K') | \mu_{t|t}^z, \mu_{t|t}^q] \quad (68)$$

$$\text{Enter if } V_{en}(a, \mu_{t|t-1}^z, \mu_{t|t-1}^q; w, K) \geq pc_e \quad (69)$$

$$\ln K' = \kappa_0^0 + \kappa_1^0 \ln K + \kappa_2^0 \mu_{t|t}^z \quad (70)$$

$$(71)$$

Given the beginning of period distributions of incumbents  $\Gamma(n_{-1}, k, x)$  and potential entrants  $\Omega(a, \mu_{t|t-1}^z, \mu_{t|t-1}^q)$ , for a proposed pair of  $(p, w)$ , by aggregating the optimal decisions of incumbents and potential entrants, we can calculate the supply of consumption that is given by the right hand side of (28) and the demand for labor that is given by the right hand side of (27) in Section 3.4. On the other hand, given that  $p$  is marginal utility of consumption ( $p = C^{-\sigma_c}$ ), it implies consumption demand from the household.  $w$  together with  $p$  also, by the household intratemporal Euler equation ( $\gamma N^{1/\sigma_n} = wC^{-\sigma_c}$ ), imply labor supply from the household. Then each period, we search pairs of  $(p, w)$  that clear both the goods and labor market. We find  $(p, w)$  by nesting the Brent's method. Once market clearing  $(p, w)$  are found, we update the incumbent distribution over  $(n_{-1}, k, x)$  and the potential entrant distribution over  $(a, \mu_{t|t-1}^z, \mu_{t|t-1}^q)$  using optimal decision rules under market clearing  $(p, w)$ .

3. Starting from the steady state distribution, we generate 500 periods of  $\{K_t, p_t, w_t\}_{t=1}^{500}$ . After discarding the first 100 observation, using OLS on the simulated data we can get new values of:

$$\{(\kappa_0, \kappa_1, \kappa_2), (a_0, a_1, a_2), (b_0, b_1, b_2), (\beta_{w,c}, \beta_{w,z}, \sigma_w), (\beta_{K,c}, \beta_{K,z}, \sigma_K)\}$$

If the new values are close enough to the previous values then we have a consistent law of motion for the aggregate capital stock, price forecasting rules, and projection equations used as the measurement equation in potential entrants' Kalman filter problem. Otherwise, update the set of coefficients and repeat the stochastic simulation.

## B.2. Accuracy of Forecasting Rules

As we described in Section 3.5 and 5.1, because of the interaction between incumbent plants' forward-looking factor demand problem and potential entrants' learning from the market clearing wage, equilibrium dynamics become history dependent in the baseline model. It

turned out that approximating the whole distribution by the aggregate capital stock alone cannot fully capture the equilibrium dynamics of the baseline model economy.

1. Forecasting rules from the baseline model economy:

$$\ln K_{t+1} = 0.16 + 0.83 \ln K_t + 0.36z_t, \quad R^2 = 0.989 \quad (72)$$

$$\ln w_t = 0.06 + 0.63 \ln K_t + 0.69z_t, \quad R^2 = 0.973 \quad (73)$$

$$\ln p_t = 0.99 - 0.81 \ln K_t - 0.34z_t, \quad R^2 = 0.994 \quad (74)$$

2. Forecasting rules from the full information economy:

$$\ln K_{t+1} = 0.12 + 0.87 \ln K_t + 0.37z_t, \quad R^2 = 0.999 \quad (75)$$

$$\ln w_t = 0.23 + 0.46 \ln K_t + 0.74z_t, \quad R^2 = 0.998 \quad (76)$$

$$\ln p_t = 0.74 - 0.54 \ln K_t - 0.47z_t, \quad R^2 = 0.999 \quad (77)$$

Comparing (73) and (76) makes it clear that in the baseline model, particularly the labor market equilibrium dynamics cannot be fully captured by tracking the aggregate capital stock alone. Instead of including higher order moments as an additional aggregate state variable to improve the precision of the price forecast, we keep the bounded rationality so that potential entrants' ability to learn about aggregate productivity from the equilibrium wage is limited.

Regarding how much potential entrants can learn about  $z_t$  using  $w_t$  and  $K_t$ , what matters is not the  $R^2$  in (73) but the fraction of variation in  $z_t$  that can be explained by  $w_t$  and  $K_t$ . In that sense, the more informative statistic regarding how much potential entrants can learn is the  $R^2$  from the projection of  $z_t$  onto a constant,  $\ln w_t$ , and  $\ln K_t$ . That value is 0.92.

## C. Pareto Distributed Operating Cost

As we have mentioned in Section 4, our choice that the stochastic operating cost comes from a log-normal distribution is arbitrary. We want to make sure that our main quantitative results do not depend on this log-normal assumption. In that regard, in this section we will provide results from the model economy where the operating cost comes from a Pareto distribution.

$$Pr(X > x) = \begin{cases} \left(\frac{\kappa}{x}\right)^\nu, & x \geq \kappa; \\ 1, & x < \kappa. \end{cases} \quad (78)$$

We recalibrate using same targets presented in Section 4.

Table 25: Endogenous Parameter Values

Description	Symbol	Value
Entry Cost	$c_e$	0.30
Operating Cost(Scale)	$\mu_{c_f}$	0.02
Operating Cost(Tail Index)	$\sigma_{c_f}$	1.51
Mean of Potential Idio.	$\bar{q}$	-0.45
S.D. of Potential Idio.	$\sigma_q$	0.11
Entrant's Capital	$k_{en}$	0.27
Resale Value of Capital	$p_s$	0.87
Labor Adj. Cost-Hiring	$c_p$	0.11
Labor Adj. Cost-Firing	$c_n$	0.11
Measure of Entrants	$M$	39.70
Disutility from work	$\gamma$	5.02

Table 26: Target Moments

Moment	Target	Model
Entry Rate	6.1%	6.0%
Exit Rate	6.1%	6.0%
Relative Size of En.	56%	56%
Relative Size of Ex.	46%	46%
Relative Prod. of En.	94%	95%
Relative Prod. of Ex.	85%	80%
Avg. $\frac{i}{k}$	12.2%	9.4%
Positive Spike( $\frac{i}{k} > 0.2$ )	18.6%	15.8%
Negative Spike( $\frac{i}{k} < -0.2$ )	1.8%	0.9%
Hours	33%	33%
Job Reallocation Rate	24%	26%

The cyclical behavior of entry and exit from the model economies with learning and full information using these new parameters are given in the following tables. In general, the assumption of a Pareto distribution versus a lognormal distribution makes little to no difference in almost all cases.

Table 27: Correlation with output

	Entry Rate	Exit Rate
Data	0.37	-0.04
Baseline	0.49	-0.02
Full Info	0.51	-0.48

Table 28: Magnitude of Fluctuations

	Entry Rate(%)		Exit Rate(%)	
	Boom	Rec.	Boom	Rec.
Data	8.1	3.4	5.8	5.1
Baseline	7.6	4.2	5.6	5.5
Full Info	6.2	5.7	5.9	6.0