Fiscal Decentralization and Intergovernmental Debt Management

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Abstract

This paper investigates how government debt is allocated between the central government and local governments among countries with various type of decentralization. For example, in Euro area, the Stability and Growth Pact provides convergence criteria on gross government debt (lower than 60% of GDP or approaching that value). Member countries decide allocation of the gross government debt between the central (federal) government and local (state) governments to achieve the criteria. On the other hand, local (state) governments do not directly accept responsibility to attain the criteria of government debt. Depending on fiscal decentralization, the central and local governments cannot cooperate to restrain government debt, because they pursue their ends independently.

We analyze theoretically and numerically intergovernmental structure and debt management of the central and local governments. We develop a theoretical model of intergovernmental financing with government debt and public investment, and explain allocation of government debt between the central and local governments in a multi-government setting.

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1. Introduction

In this paper we examine how government debt is allocated between the central government and local governments among countries with various type of decentralization. Namely, this paper will analyze theoretically and numerically intergovernmental structure and debt management of the central and local governments by developing a simple game between the two governments with the overlapping tax bases between them.

Generally, the central (or federal) government and local (and/or state) governments can decide to borrow money independently in their budget making processes. On the other hand, there are some fiscal rules or restrictions on borrowing funds (or issuing government bonds) in industrial countries. For example, in Euro area, the Stability and Growth Pact provides convergence criteria on gross government debt (lower than 60% of GDP or approaching that value). Member countries decide allocation of the gross government debt between the central (federal) government and local (state) governments to achieve the criteria. These restrictions are needed for maintaining fiscal sustainability.

On the other hand, local (state) governments do not directly accept responsibility to attain the criteria of government debt. Depending on fiscal decentralization, the central and local governments cannot cooperate to restrain government debt, because they pursue their ends independently.

Figure 1 shows government debt to GDP ratio by sub-sector of general government. In countries of unitary system, government debt of local government tends to be much less than one of the central governments. In countries of federal system, though debt of state and local governments tends to be less than one of federal government, the degree in federal states is not so much as one in unitary states. Usually, intergovernmental system in federal states is more decentralized than one in unitary system. Is this tendency shown in Figure 1 affected by fiscal decentralization?

We analyze theoretically and numerically intergovernmental structure and debt management of the central and local governments. We develop a theoretical model of intergovernmental financing with government debt and public investment, and explain allocation of government debt between the central and local governments in a multi-government setting.

In this paper without incorporating any uncertainty or imperfect information of effort with respect to public investment and other government activities, we develop a simple game theoretic model of the central and local governments, depending on the behavior of the central government with respect to intergovernmental transfers and tax structure.

We pay attention to the vertical externality of shared tax bases between the central and local governments. Multileveled government normally means some commonality of tax base between central and local governments. As a result the tax base may overlap and shared tax bases create another type of common pool problem. It is now well recognized that vertical externalities are likely to leave local taxes too high. This is because each local government unduly discounts the pressure on central government's spending it creates by raising its own tax rate. See Keen and Kotsogiannis (2002), Keen (1998), and Wilson (1999) among others. In this paper we do not consider such vertical/horizontal tax competition between central and local governments and would simply assume that tax rates are given for central and local governments. Rather, we would like to focus on another inefficiency of local expenditures due to overlapping tax bases.

By assuming that the tax share is exogenously given, we incorporate two sources of inefficiency. First, the distribution of public spending between the central and local governments is not necessarily determined optimally. If the tax share to the central government is too high, the size of local public spending is too low (and vice versa). Second, local public investment may have a positive vertical externality effect. Namely, if an increase in local expenditure on infrastructure stimulates macroeconomic activities, it may enlarge the overlapping tax base, which would then increase taxes for the central government at the given share of tax base between two governments. This is a positive spillover of vertical externality. In this sense, the non-cooperative Nash equilibrium level of local public investment is too low.

This paper consists of five sections. In Section 2, we develop a theoretical model of the central and local governments with vertical externality of overlapping tax bases. Then we formulate a theoretical framework to compare intergovernmental systems between federal and unitary states in Section 3. In Section 4, we investigate debt management of both central and local governments using numerical analysis, by specify functional forms in the above model. Finally, we present some concluding remarks in Section 5.

2. Analytical Model of the Central and Local Governments

2.1 Model

We develop a two-period intergovernmental financing model of two governments, the central government (or CG), the lower-level local government (or LG) in a small open economy. For simplicity, we consider the representative local government, and do not consider the free-riding and/or spillover effects within local governments. This is just an assumption for simplicity. There are many papers to explore the horizontal and vertical externalities due to non-cooperative competition among local governments. See Wilson (1999) among others. As shown in Appendix, the analytical results would be qualitatively the same even if we consider non-cooperative behavior of multi-local governments.

The representative local government (LG) provides local public goods g_t, and the central government (CG) provides nation-wide public goods G_t in each period. Each public good is beneficial and its utility is given by a twice-continuously differentiable and strictly quasi-concave function. Moreover, we assume that all goods are normal ones. The relative price of each good is set to be unity for simplicity.

Both the central and local governments levy taxes on overlapping economic activities. Since the tax base is overlapping, the tax revenue may be shared by the two governments. We set τ_L as local government's tax rate, $0<\tau_L<1$. τ_C denotes the tax rate of the central government, $0<\tau_C<1$. The tax rates τ_C and τ_L are assumed to be fixed over time once they are decided.

Thus, the social welfare W, which reflects the representative agent's preferences over public goods, is given by

$$W = u(G_1) + v(g_1) + \delta\{u(G_2) + v(g_2)\} \tag{1}$$

where $0 < \delta < 1$ is a discount factor. For simplicity, private consumption is assumed to be fixed and hence we only consider the utility from public goods.

The local government also conducts public investment k in period 1, which has a productive effect of raising tax revenue in period 2. Let Y_t represent common tax base of the two governments in period t (t = 1, 2). We assume that Y_1 is exogenously given but Y_2 is dependent on public works conducted by the local government in period 1 and excess burden with taxation. $Y_2 = Y_1 + f(k) - EB(\tau_C, \tau_L)$. EB denotes excess burden with taxation of the central government and local government. $EB_C \equiv \dfrac{\partial EB(\bullet)}{\partial \tau_C}$ may not be the same as $EB_L \equiv \dfrac{\partial EB(\bullet)}{\partial \tau_L}$. It means that magnitude of tax distortion of the national tax may be different from one of the local tax, because the national tax structure may not be the same as local tax structure. Also the function of excess burden with taxation, $EB_j(\bullet)$ satisfies the following conditions: $EB_j' > 0$, $EB_j'' > 0$; $j = C, L$.

k denotes local public investment in period 1, which would increase total tax revenue of period 2. Investment product function $f(\bullet)$ satisfies the standard Inada condition: $f' > 0$, $f'' < 0$. For simplicity we do not consider public investment by the central government. In this paper we consider nation-widely beneficial local public investment. In a multi-local government setting local public investment does not have spillover effects over regions. Still it has the vertical externality effect on the central government's tax revenue.

Next, we specify each government's budget constraint. The period-by-period budget constraints of CG are given as follows,

$$B = G_1 + Z_1 - \tau_C Y_1 \tag{2-1}$$

$$G_2 + Z_2 + (1+r)B = \tau_C Y_2 \tag{2-2}$$

where Z_t is grants from the central government to the local government in period t. B is the central government debt. $r > 0$ is the exogenously given world interest rate.

The period-by-period budget constraints of LG are given as follows,

$$D = g_1 + k - Z_1 - \tau_L Y_1 \tag{3-1}$$

$$g_2 + (1+r)D = Z_2 + \tau_L Y_2 \tag{3-2}$$

where D is the local government debt.

From (2) and (3) we can rewrite the intertemporal budget constraints of the central and local government, respectively, as follows.

$$G_1 + \frac{G_2}{1+r} = \tau_C Y_1 + \frac{\tau_C Y_2}{1+r} - Z_1 - \frac{Z_2}{1+r} \tag{2-3}$$

$$g_1 + \frac{g_2}{1+r} + k = \tau_L Y_1 + \frac{\tau_L Y_2}{1+r} + Z_1 + \frac{Z_2}{1+r} \tag{3-3}$$

2.2 Consolidated Government

First of all, we investigate the Pareto efficient first best allocation in this model as a benchmark. Since we do not incorporate any uncertainty or imperfect information with respect to public investment and other government activities, the consolidated government, consolidating CG and LG, could attain the first best by allocating optimally the total tax revenues among nation-wide public goods and local public goods in each period. Namely, the consolidated government, that implements the optimal allocation $\{G_t, g_t, k, \tau\}$, maximizes social welfare (1) subject to the following overall feasibility constraint

$$\tau Y_1 + \frac{\tau Y_2}{1+r} = G_1 + \frac{G_2}{1+r} + g_1 + \frac{g_2}{1+r} + k \tag{4}$$

which is obtained from (2-3) and (3-3) by eliminating Z_1 and Z_2. Now, Y_2 = $Y_1 + f(k) - EB(\tau)$.

First order conditions of this optimization problem are as follows,

$$u_{G1} - \mu = 0$$

$$\delta u_{G2} - \frac{\mu}{1+r} = 0 \qquad \text{where } u_{Gt} \equiv \frac{\partial u(G_t)}{\partial G_t}$$

$$v_{g1} - \mu = 0$$

$$\delta v_{g2} - \frac{\mu}{1+r} = 0 \qquad \text{where } v_{gt} \equiv \frac{\partial v(g_t)}{\partial g_t}$$

$$\mu \left\{ \frac{\tau f'(k)}{1+r} - 1 \right\} = 0$$

$$\mu\left\{Y_1+\frac{Y_2}{1+r}-\frac{\tau EB'}{1+r}\right\}=0 \qquad \text{where } \; EB' \equiv \frac{\partial EB(\tau)}{\partial \tau}$$

μ is the Lagrangian multiplier of equation (4). From these conditions we have

$$v_{g1} = u_{G1} \tag{5-1}$$

$$u_{G2} = v_{g2} \tag{5-2}$$

$$\frac{u_{G1}}{u_{G2}} = \frac{v_{g1}}{v_{v2}} = (1+r)\delta \tag{5-3}$$

$$\tau f'(k) = 1+r \tag{5-4}$$

$$\tau EB' = (2+r)Y_1 + f(k) - EB(\tau) \tag{5-5}$$

The above optimality conditions (5-1,.,5) and the feasibility condition (4) determine the Pareto efficient allocation as the benchmark case. Conditions (5-1) and (5-2) mean that the marginal benefit of pubic goods is equalized between CG and LG. Condition (5-3) governs the standard (intertemporal) optimal allocation of public spending between two periods. Finally, condition (5-4) is the standard first-best criterion of public investment.

In this situation, allocation between the central government debt, B, and the local government debt, D, is neutral for social welfare.

3. Theoretical Analyses

3.1 Comparing Intergovernmental Systems between Federal and Unitary States

In this section, we establish a theoretical framework to compare intergovernmental systems between federal and unitary states, based on the above model in section 2. The system in federal states is more decentralized than that in unitary states. As we will formulate our models, we assume that the central and local governments decided their policies independently under the setting of federal states.. Also we assume that the central government is the leader and the local government is the follower under the setting of unitary states. Namely, at the first stage the central government determines its policies, and then at the second stage local government determines its policies.

Before describing our model, we formulate the intergovernmental transfer system in our model. In fact, there are many intergovernmental transfers in most countries. Since formula of intergovernmental transfers vary in each country and complicated, it is not so easy to specify this formula in this model. Therefore, we formulate generally as follows,.

$$Z_1 = Z_1(g_1, k) \tag{6-1}$$

$$Z_2 = Z_2(g_2) \tag{6-2}$$

We assume that intergovernmental transfer in period 1 is a function of g_1 and k, and intergovernmental transfer in period 2 is a function of g_2. Usually, the following conditions seem to be satisfied; $Z_{1g} \equiv \frac{\partial Z_1}{\partial g_1} > 0$, $Z_{1k} \equiv \frac{\partial Z_1}{\partial k} > 0$, and $Z_{2g} \equiv \frac{\partial Z_2}{\partial g_2} > 0$.

3.2 Situation in Federal States without Bond Issuance Cap

Suppose first of all the fully (or isolated) decentralized Nash equilibrium.

We assume that both CG and LG are benevolent. It means that both governments maximize the social welfare, (1). CG maximizes (1) subject to (2-3), including (6-1), and (6-2), by choosing nation wide public goods while assuming local public goods fixed. Similarly, LG maximizes (1) subject to (3-3), including (6-1), and (6-2), by choosing local public goods and investment, while assuming nation-wide public goods fixed. In this section, $Y_2 = Y_1 + f(k) - EB(\tau_C, \tau_L)$, because the central and local government decide their tax rates independently. Then, first order conditions of this Nash non-cooperative equilibrium are as follows,

$$u_{G1} - \psi_C = 0$$

$$\delta u_{G2} - \frac{\psi_C}{1+r} = 0$$

$$\psi_C \left\{ Y_1 + \frac{Y_2}{1+r} - \frac{\tau_C EB_C}{1+r} \right\} = 0$$

$$v_{g1} - \psi_L (1 - Z_{1g}) = 0$$

$$\delta v_{g2} - \frac{\psi_L}{1+r}(1 - Z_{2g}) = 0$$

$$\psi_L \left\{ \frac{\tau_L f'(k)}{1+r} - (1 - Z_{1k}) \right\} = 0$$

$$\psi_L \left\{ Y_1 + \frac{Y_2}{1+r} - \frac{\tau_L EB_L}{1+r} \right\} = 0$$

where ψ_C and ψ_L are the Lagrangian multipliers of equations (2-3) and (3-3) respectively. From these conditions we have

$$\frac{u_{G1}}{u_{G2}} = (1+r)\delta \qquad (7\text{-}1)$$

$$\frac{v_{g1}}{v_{g2}} = \frac{1 - Z_{1g}}{1 - Z_{2g}}(1+r)\delta = \frac{1 - Z_{1g}}{1 - Z_{2g}} \frac{u_{G1}}{u_{G2}} \qquad (7\text{-}2)$$

$$f'(k) = \frac{1 - Z_{1k}}{\tau_L}(1+r) \qquad (7\text{-}3)$$

$$\tau_C EB_C = \tau_L EB_L = (2+r)Y_1 + f(k) - EB(\tau_C, \tau_L) \qquad (7\text{-}4)$$

Condition (7-1), which is the same as (5-3), implies that relative (intertemporal) allocation between G_1 and G_2 is efficient. But the levels of these public goods and local investment are not necessarily provided optimally. In other words, conditions (5-1) and (5-2) do not necessarily hold since the total levels of pubic goods, $G_1 + \frac{G_2}{1+r}$ and $g_1 + \frac{g_2}{1+r}$, are arbitrarily set, depending on the exogenous parameter, Z_{1g} and Z_{2g}.

Moreover, (7-3) means that k is under-provided due to the overlapping tax base when $(1 - Z_{1k})/\tau_L > \tau$. Optimality condition for k, (5-4), is not always held. Since the local government does not take into account the positive spillover effect of increasing the overlapping tax base on public goods provided by the central government, local public investment provided by the local government is not sufficient and total tax revenue shared by both governments in period 2 is inefficiently low.

3.3 Situation in Unitary States without Bond Issuance Cap

In order to illustrate intergovernmental system in a unitary state, we assume CG is the leader and LG is the follower. The game is done at the beginning of period 1. Namely, at the first stage CG determines provision of nation-wide public goods, G_1 and G_2, tax rate, τ_C, and scheme of intergovernmental transfer with regarding g_1, g_2, and k fixed, and then at the second stage LG determines its expenditures and tax rate, τ_L.

We first investigate the behavior of LG, which occurs at the beginning of the first period. LG regards nation-wide public goods as fixed when LG maximizes social welfare (1) subject to (3-3), including (6-1), and (6-2), at given levels of policy variables of the central government.

$$v_{g1} - \psi_L(1 - Z_{1g}) = 0$$

$$\delta v_{g2} - \frac{\psi_L}{1+r}(1 - Z_{2g}) = 0$$

$$\psi_L \left\{ \frac{\tau_L f'(k)}{1+r} - (1 - Z_{1k}) \right\} = 0$$

$$\psi_L \left\{ Y_1 + \frac{Y_2}{1+r} - \frac{\tau_L EB_L}{1+r} \right\} = 0$$

where ψ_L is the Lagrangian multiplier of equation (3-3). From these conditions we have

$$\frac{v_{g1}}{v_{g2}} = \frac{1 - Z_{1g}}{1 - Z_{2g}}(1+r)\delta \qquad (7\text{-}2)$$

$$f'(k) = \frac{1 - Z_{1k}}{\tau_L}(1+r) \qquad (7\text{-}3)$$

$$\tau_L EB_L = (2+r)Y_1 + f(k) - EB(\tau_C, \tau_L) \quad (7\text{-}4')$$

If Z_{1k} is a function of just k, from (7-3) we find that k is a function of only τ_L. From (7-5'), therefore, τ_L depends on τ_C. Hence k depends on τ_C in this situation. If Z_{1g} is a function of just g_1, we find that g_2 is as function of only g_1. Totally, from (3-3) and (7-2), it suggests that g_1 depends on τ_C. For simplicity, we presume that Z_{1k} is a function of just k, and Z_{1g} is a function of just g_1.

At the first stage, CG maximizes (1) subject to (2-3), including (6-1), and (6-2), by choosing nation wide public goods and tax rate with response functions of the local government. That is,

$$\max W = u(G_1) + v[g_1(\tau_C)] + \delta\{u(G_2) + v[g_2(\tau_C)]\} \quad (1')$$

$$\text{s.t.} \quad \begin{aligned} G_1 + \frac{G_2}{1+r} = \tau_C Y_1 + \frac{\tau_C}{1+r}\{Y_1 + f[k(\tau_C)] - EB[\tau_C, \tau_L(\tau_C)]\} \\ - Z_1[g_1(\tau_C), k(\tau_C)] - \frac{Z_2[g_2(\tau_C)]}{1+r} \end{aligned} \quad (2\text{-}3')$$

Then, first order conditions of this CG's optimization problem are as follows,

$$u_{G1} - \hat{\psi}_C = 0$$

$$\delta u_{G2} - \frac{\hat{\psi}_C}{1+r} = 0$$

$$\begin{aligned} v_{g1}\frac{\partial g_1}{\partial \tau_C} + v_{g2}\frac{\partial g_2}{\partial \tau_C} + \hat{\psi}_C\left\{Y_1 + \frac{Y_2}{1+r} + \frac{\tau_C}{1+r}\left(f'\frac{\partial k}{\partial \tau_C} - EB_C - EB_L\frac{\partial \tau_L}{\partial \tau_C}\right)\right\} \\ -\hat{\psi}_C\left\{Z_{1g}\frac{\partial g_1}{\partial \tau_C} + Z_{1k}\frac{\partial k}{\partial \tau_C} + \frac{Z_{2g}}{1+r}\frac{\partial g_2}{\partial \tau_C}\right\} = 0 \end{aligned}$$

where $\hat{\psi}_C$ is the Lagrangian multiplier of equation (2-3'). From these conditions, using (7-2), we have

$$\frac{u_{G1}}{u_{G2}} = (1+r)\delta \quad (7\text{-}1)$$

$$\begin{aligned} & v_{g1}\left\{\frac{\partial g_1}{\partial \tau_C} + \frac{1-Z_{2g}}{(1-Z_{1g})(1+r)\delta}\frac{\partial g_2}{\partial \tau_C}\right\} \\ & = u_{G1}\left[\left\{Z_{1g}\frac{\partial g_1}{\partial \tau_C} + Z_{1k}\frac{\partial k}{\partial \tau_C} + \frac{Z_{2g}}{1+r}\frac{\partial g_2}{\partial \tau_C}\right\}\right. \\ & \qquad \left. - \left\{Y_1 + \frac{Y_2}{1+r} + \frac{\tau_C}{1+r}\left(f'\frac{\partial k}{\partial \tau_C} - EB_C - EB_L\frac{\partial \tau_L}{\partial \tau_C}\right)\right\}\right] \end{aligned} \quad (8\text{-}1)$$

Condition (7-1) is the same as an optimality condition under federal states in section 3.2. However, condition (8-1) for τ_C is usually different from one under federal states.

3.4 Situation in Federal States with Bond Issuance Cap

We examine effects of bond issuance cap. First, we presume that bond issuance cap of LG, $\bar{D}$, is set exogenously. LG chooses g_1, g_2, k and τ_L at given level of $\bar{D}$. Under the model of federal states, CG decides G_1, G_2 and τ_C at the same time. CG is assumed to impose no bond issuance cap.

LG maximizes the social welfare (1) subject to budget constraint with bond issuance cap at given levels of policy variables of the central government.

$$\max W = u(G_1) + v(g_1) + \delta\{u(G_2) + v(g_2)\} \tag{1}$$

$$\text{s.t. } \bar{D} = g_1 + k - Z_1(g_1, k) - \tau_L Y_1 \tag{3-1'}$$

$$g_2 + (1+r)\bar{D} = Z_2(g_2) + \tau_L Y_2 \tag{3-2'}$$

The first order condition of this maximization is given as follows,

$$v_{g1} - \lambda_{L1}(1 - Z_{1g}) = 0$$

$$\delta v_{g2} - \lambda_{L2}(1 - Z_{2g}) = 0$$

$$-\lambda_{L1}(1 - Z_{1k}) + \lambda_{L2}\tau_L f'(k) = 0$$

$$\lambda_{L1} Y_1 + \lambda_{L1}(Y_2 - \tau_L EB_L) = 0$$

where λ_{L1} and λ_{L2} are the Lagrangian multipliers of equations (3-1') and (3-2'), respectively. From these conditions we have

$$\frac{v_{g1}}{v_{g2}} = \frac{1 - Z_{1g}}{1 - Z_{2g}} \frac{\tau_L f' \delta}{1 - Z_{1k}} \tag{9-1}$$

$$Y_1 + \frac{1 - Z_{1k}}{\tau_L f'}\{Y_1 + f(k) - EB(\tau_C, \tau_L) - \tau_L EB_L\} = 0 \tag{9-2}$$

If Z_{1k} is a function of just k, from (9-2) we find that k is a function of only τ_L at given level of τ_C. Hence, from (3-1'), it suggests that g_1 is a function of τ_L at given levels of τ_C and $\bar{D}$. If Z_{1g} is a function of just g_1, we can solve g_2 and τ_L from (3-2') and (9-1) in this Nash non-cooperative equilibrium between CG and LG.

On the other hand, CG maximizes the social welfare (1) under the budget constraint (2-3) by choosing G_1, G_2, tax rate, τ_C, and national bonds B. The first order condition of this maximization is given as follows, similar in section 3.2,

$$\frac{u_{G1}}{u_{G2}} = (1+r)\delta \tag{7-1}$$

$$\tau_C EB_C = (2+r)Y_1 + f(k) - EB(\tau_C, \tau_L) \tag{7-4''}$$

In this situation, values of G_1, G_2, τ_C, g_1, g_2, k, and τ_L in this Nash non-cooperative equilibrium satisfies equations (2-3), (3-1'), (3-2'), (7-1), (7-4''), (9-1), and (9-2).

3.5 Situation in Unitary States with Bond Issuance Cap

In this section we analyze effects of bond issuance cap in unitary states. Like the previous section, we presume that bond issuance cap of LG, $\bar{D}$, at the first stage is set exogenously. CG is the leader and LG is the follower. The game is done at the beginning of period 1. Namely, at the first stage CG determines provision of nation-wide public goods, G_1 and G_2, tax rate, τ_C, and scheme of intergovernmental transfer with regarding g_1, g_2, and k fixed. At the second stage, LG chooses g_1, g_2, k and τ_L at given levels of $\bar{D}$.

At the second stage, LG maximizes the social welfare (1) subject to the following budget constraints with bond issuance cap.

$$\max W = u(G_1) + v(g_1) + \delta\{u(G_2) + v(g_2)\} \tag{1}$$

$$\text{s.t.} \quad \bar{D} = g_1 + k - Z_1(g_1, k) - \tau_L Y_1 \tag{3-1'}$$

$$g_2 + (1+r)\bar{D} = Z_2(g_2) + \tau_L Y_2 \tag{3-2'}$$

The first order condition of this maximization is given as follows,

$$\frac{v_{g1}}{v_{g2}} = \frac{1-Z_{1g}}{1-Z_{2g}} \frac{\tau_L f'\delta}{1-Z_{1k}} \tag{9-1}$$

$$Y_1 + \frac{1-Z_{1k}}{\tau_L f'}\{Y_1 + f(k) - EB(\tau_C, \tau_L) - \tau_L EB_L\} = 0 \tag{9-2}$$

These conditions are the same as those in section 3.4.

If Z_{1k} is a function of just k, from (9-2') we find that k is a function of τ_C and τ_L. Hence, from (3-1'), it suggests that g_1 is a function of τ_C and τ_L. From (3-1'), we find that g_2 is a function of τ_C and τ_L. Thus if Z_{1g} is a function of just g_1, we can solve τ_L as a function of τ_C from (9-1). Totally, we find that g_1, g_2 and k are all functions of τ_L. However, these functions are different from those in section 3.3, because the first order conditions in this section, (9-1) and (9-2), are different from those in section 3.3, (7-2), (7-3), and (7-4'). Therefore, we describe these functions in this section as $\hat{g}_1(\tau_C), \hat{g}_2(\tau_C), \hat{k}(\tau_C)$ and $\hat{\tau}_L(\tau_C)$.

At the first stage, CG maximizes (1) subject to (2-3), including (6-1), and (6-2), by choosing nation wide public goods and tax rate with response functions of the local government. That is,

$$\max W = u(G_1) + v[\hat{g}_1(\tau_C)] + \delta\{u(G_2) + v[\hat{g}_2(\tau_C)]\} \tag{1''}$$

$$\text{s.t.} \quad \begin{aligned} G_1 + \frac{G_2}{1+r} = \tau_C Y_1 + \frac{\tau_C}{1+r}\Big\{Y_1 + f[\hat{k}(\tau_C)] - EB[\tau_C, \hat{\tau}_L(\tau_C)]\Big\} \\ - Z_1[\hat{g}_1(\tau_C), \hat{k}(\tau_C)] - \frac{Z_2[\hat{g}_2(\tau_C)]}{1+r} \end{aligned} \tag{2-3'}$$

Then, first order conditions of this CG's optimization problem are as follows,

$$u_{G1}-\lambda_C=0$$

$$\delta u_{G2}-\frac{\lambda_C}{1+r}=0$$

$$v_{g1}\frac{\partial\hat{g}_1}{\partial\tau_C}+v_{g2}\frac{\partial\hat{g}_2}{\partial\tau_C}+\lambda_C\left\{Y_1+\frac{Y_2}{1+r}+\frac{\tau_C}{1+r}\left(f'\frac{\partial\hat{k}}{\partial\tau_C}-EB_C-EB_L\frac{\partial\hat{\tau}_L}{\partial\tau_C}\right)\right\}$$
$$-\lambda_C\left\{Z_{1g}\frac{\partial\hat{g}_1}{\partial\tau_C}+Z_{1k}\frac{\partial\hat{k}}{\partial\tau_C}+\frac{Z_{2g}}{1+r}\frac{\partial\hat{g}_2}{\partial\tau_C}\right\}=0$$

where λ_C is the Lagrangian multiplier of equation (2-3'). From these conditions, using (9-1), we have

$$\frac{u_{G1}}{u_{G2}}=(1+r)\delta \tag{7-1}$$

$$v_{g1}\left\{\frac{\partial\hat{g}_1}{\partial\tau_C}+\frac{(1-Z_{2g})(1-Z_{1k})}{(1-Z_{1g})\tau_L f'\delta}\frac{\partial\hat{g}_2}{\partial\tau_C}\right\}$$
$$=u_{G1}\left[\left\{Z_{1g}\frac{\partial\hat{g}_1}{\partial\tau_C}+Z_{1k}\frac{\partial\hat{k}}{\partial\tau_C}+\frac{Z_{2g}}{1+r}\frac{\partial\hat{g}_2}{\partial\tau_C}\right\}\right. \tag{10-1}$$
$$\left.-\left\{Y_1+\frac{Y_2}{1+r}+\frac{\tau_C}{1+r}\left(f'\frac{\partial\hat{k}}{\partial\tau_C}-EB_C-EB_L\frac{\partial\hat{\tau}_L}{\partial\tau_C}\right)\right\}\right]$$

Condition (7-1) is the same as an optimality condition under federal states in section 3.4. However, condition (10-1) for τ_C is usually different from one in section 3.4.

4. Numerical Analysis

4.1 Specification of the Model

We investigate debt management of both central and local governments using numerical analysis. Before the analysis, we specify functional forms in the above model.

The social welfare function is specified as

$$W=\frac{G_1^{1-\zeta}-1}{1-\zeta}+\frac{g_1^{1-\theta}-1}{1-\theta}+\delta\left\{\frac{G_2^{1-\zeta}-1}{1-\zeta}+\frac{g_2^{1-\theta}-1}{1-\theta}\right\} \qquad 0<\delta<1 \tag{11}$$

The production function of public investment is specified as

$$f(k)=Ak^{\alpha}$$

The functions of excess burden of taxation are specified as

$$EB(\tau_C,\tau_L)=\Omega(\omega_C\tau_C+\omega_L\tau_L)^2 \qquad \text{where } \Omega>0,\ \omega_j>0 \quad j=C,L \tag{12}$$

Moreover, we formulate the intergovernmental transfer system as follows,

$$Z_1 = m_1 g_1 + m_k k \tag{13-1}$$

$$Z_2 = m_2 g_2 \tag{13-2}$$

We presume that m_1, m_2, and m_k are positive constants and set exogenously.

4.2 First Best Solution

In these specifications, the optimality conditions of the first best solution in this model, as described in section 2.2, are as follows.

$$g_1^{-\theta} = G_1^{-\zeta} \tag{14-1}$$

$$g_2^{-\theta} = G_2^{-\zeta} \tag{14-2}$$

$$\left(\frac{g_1}{g_2}\right)^{-\theta} = \left(\frac{G_1}{G_2}\right)^{-\zeta} = (1+r)\delta \tag{14-3}$$

$$\tau A \alpha k^{\alpha-1} = 1+r \tag{14-4}$$

$$3\Omega\tau^2 = (2+r)Y_1 + Ak^{\alpha} \tag{14-5}$$

where $EB(\tau) = \Omega\tau^2$. From (14-4), we obtain

$$k \equiv \left(\frac{A\alpha\tau}{1+r}\right)^{\frac{1}{1-\alpha}}$$

Then from (14-5), we obtain the first best solution of τ from solving

$$3\Omega\tau^2 = (2+r)Y_1 + A\left(\frac{A\alpha\tau}{1+r}\right)^{\frac{\alpha}{1-\alpha}},$$

The solution of τ the above equation is denoted by τ^*. Also we have the first best solution of k as

$$k^* \equiv \left(\frac{A\alpha\tau^*}{1+r}\right)^{\frac{1}{1-\alpha}}$$

Substituting the above optimality conditions into the budget constraint, we have

$$\tau^*\left\{Y_1 + \frac{Y_2^*}{1+r}\right\} = g_1^{\theta/\zeta} + \frac{\{\delta(1+r)\}^{1/\zeta} g_1^{\theta/\zeta}}{1+r} + g_1 + \frac{\{\delta(1+r)\}^{1/\theta} g_1}{1+r} + k^*$$

where $Y_2^* = Y_1 + A(k^*)^{\alpha} - \Omega(\tau^*)^2$. The solution of g_1 in the above equation is the first best solution of g_1, denoted by g_1^*. Also substituting g_1^* into the optimality conditions, we get

$$G_1^* \equiv \left(g_1^*\right)^{\theta/\zeta}$$

$$G_2^* \equiv \{\delta(1+r)\}^{1/\zeta}\left(g_1^*\right)^{\theta/\zeta}$$

$$g_2^* \equiv \{\delta(1+r)\}^{1/\theta} g_1^*$$

4.3 Situation in Federal States without Bond Issuance Cap

In the solution of situation in federal states without bond issuance cap, as explained in section 3.2, the optimality conditions of LG are as follows

$$A\alpha k^{\alpha-1} = \frac{(1-m_k)(1+r)}{\tau_L} \tag{15-1}$$

$$\left(\frac{g_1}{g_2}\right)^{-\theta} = \frac{1-m_1}{1-m_2}(1+r)\delta \tag{15-2}$$

$$2\Omega\omega_L\tau_L(\omega_C\tau_C+\omega_L\tau_L) = (2+r)Y_1 + Ak^\alpha - \Omega(\omega_C\tau_C+\omega_L\tau_L)^2 \tag{15-3}$$

On the other hand, the optimality conditions of CG are as follows

$$\left(\frac{G_1}{G_2}\right)^{-\zeta} = (1+r)\delta \tag{15-4}$$

$$2\Omega\omega_C\tau_C(\omega_C\tau_C+\omega_L\tau_L) = (2+r)Y_1 + Ak^\alpha - \Omega(\omega_C\tau_C+\omega_L\tau_L)^2 \tag{15-5}$$

In this Nash equilibrium, we have

$$\omega_C\tau_C = \omega_L\tau_L \tag{15-6}$$

from (15-3) and (15-5).

From (15-1), (15-3) and (15-6), we have

$$8\Omega(\omega_L\tau_L)^2 = (2+r)Y_1 + A\left\{\frac{A\alpha\tau_L}{(1-m_k)(1+r)}\right\}^{\frac{\alpha}{1-\alpha}}$$

Solving the above equation, we obtain a solution of τ_L in this situation, denoted by τ_L^{FN}. Then substituting τ_L^{FN} into (15-1) and (15-6), we have

$$k^{FN} \equiv \left\{\frac{A\alpha\tau_L^{FN}}{(1-m_k)(1+r)}\right\}^{\frac{1}{1-\alpha}},$$

$$\tau_C^{FN} \equiv \frac{\omega_L}{\omega_C}\tau_L^{FN},$$

From (3-3) and (15-2), we obtain

$$g_1^{FN} \equiv \frac{(1+r)Y_1\tau_L^{FN} + \tau_L^{FN}Y_2^{FN} - (1+r)(1-m_k)k^{FN}}{(1-m_1)(1+r) + (1-m_2)\{(1+r)\delta\}^{\frac{1}{\theta}}}$$

where $Y_2^{FN} = Y_1 + A(k^{FN})^\alpha - \Omega(\omega_C\tau_C^{FN} + \omega_L\tau_L^{FN})^2$. Solving the above equation, we have a solution of g_1 in this situation, denoted by g_1^{FN}. Thus

$$g_2^{FN} \equiv \{\delta(1+r)\}^{1/\theta} g_1^{FN} .$$

and

$$Z_1^{FN} \equiv m_1 g_1^{FN} + m_k k^{FN}$$

$$Z_2^{FN} \equiv m_2 g_2^{FN}$$

From the above solutions, we have

$$G_1^{FN} \equiv \frac{(1+r)\tau_C^{FN} Y_1 + \tau_C^{FN} Y_2^{FN} - (1+r)Z_1^{FN} - Z_2^{FN}}{1+r+\{(1+r)\delta\}^{\frac{1}{\zeta}}}$$

and

$$G_2^{FN} \equiv \{\delta(1+r)\}^{1/\zeta} G_1^{FN} .$$

In this Nash equilibrium, government debts of the central and local governments are determined as follows

$$B^{FN} = G_1^{FN} + Z_1^{FN} - \tau_C^{FN} Y_1$$

$$D^{FN} = g_1^{FN} + k^{FN} - Z_1^{FN} - \tau_L^{FN} Y_1$$

4.4 Situation in Unitary States without Bond Issuance Cap

In situation in unitary states without bond issuance cap, as described in Section 3.3, the optimality conditions of both CG and LG are the following

$$A\alpha k^{\alpha-1} = \frac{(1-m_k)(1+r)}{\tau_L} \tag{16-1}$$

$$\left(\frac{g_1}{g_2}\right)^{-\theta} = \frac{1-m_1}{1-m_2}(1+r)\delta \tag{16-2}$$

$$2\Omega\omega_L\tau_L(\omega_C\tau_C + \omega_L\tau_L) = (2+r)Y_1 + Ak^{\alpha} - \Omega(\omega_C\tau_C + \omega_L\tau_L)^2 \tag{16-3}$$

From (16-1) and (16-3), we have

$$2\Omega\omega_L\tau_L(\omega_C\tau_C + \omega_L\tau_L) = (2+r)Y_1 + A\left\{\frac{A\alpha\tau_L}{(1-m_k)(1+r)}\right\}^{\frac{\alpha}{1-\alpha}} - \Omega(\omega_C\tau_C + \omega_L\tau_L)^2$$

Solving the above equation, we obtain a response function with respect to τ_L in this situation, denoted by $\tau_L^{UN}(\tau_C)$, as a function of τ_C. Then substituting $\tau_L^{UN}(\tau_C)$ into (15-1), we have

$$k^{UN}(\tau_C) \equiv \left\{\frac{A\alpha\tau_L^{UN}(\tau_C)}{(1-m_k)(1+r)}\right\}^{\frac{1}{1-\alpha}}$$

From (3-3) and (16-2), we obtain

$$g_1^{UN}(\tau_C) \equiv \frac{(1+r)Y_1\tau_L^{UN}(\tau_C)+\tau_L^{UN}(\tau_C)Y_2^{UN}(\tau_C)-(1+r)(1-m_k)k^{UN}(\tau_C)}{(1-m_1)(1+r)+(1-m_2)\{(1+r)\delta\}^{\frac{1}{\theta}}}$$

where $Y_2^{UN}(\tau_C)=Y_1+A\{k^{UN}(\tau_C)\}^{\alpha}-\Omega\{\omega_C\tau_C+\omega_L\tau_L^{UN}(\tau_C)\}^2$. Thus

$$g_2^{UN}(\tau_C)\equiv\{\delta(1+r)\}^{1/\theta}g_1^{UN}(\tau_C).$$

and

$$Z_1^{UN}(\tau_C)\equiv m_1g_1^{UN}(\tau_C)+m_kk^{UN}(\tau_C)$$

$$Z_2^{UN}(\tau_C)\equiv m_2g_2^{UN}(\tau_C)$$

On the other hand, the optimality conditions of CG are as follows

$$\left(\frac{G_1}{G_2}\right)^{-\zeta}=(1+r)\delta \tag{16-4}$$

$$\begin{aligned}&\left\{\frac{\partial g_1^{UN}}{\partial\tau_C}+\frac{1-m_2}{(1-m_1)(1+r)\delta}\frac{\partial g_2^{UN}}{\partial\tau_C}\right\}(g_1^{UN})^{-\theta}\\&=G_1^{-\zeta}\left[\left\{m_1\frac{\partial g_1^{UN}}{\partial\tau_C}+m_k\frac{\partial k^{UN}}{\partial\tau_C}+\frac{m_2}{1+r}\frac{\partial g_2^{UN}}{\partial\tau_C}\right\}\right.\\&\qquad-\left\{Y_1+\frac{Y_2^{UN}(\tau_C)}{1+r}+\frac{\tau_C}{1+r}\left(A\alpha(k^{UN})^{\alpha-1}\frac{\partial k^{UN}}{\partial\tau_C}\right.\right.\\&\qquad\qquad\left.\left.\left.-2\Omega\{\omega_C\tau_C+\omega_L\tau_L^{UN}(\tau_C)\}\{\omega_C+\omega_L\frac{\partial\tau_L^{UN}}{\partial\tau_C}\}\right)\right\}\right]\end{aligned} \tag{16-5}$$

From (2-3') and (16-4), we have

$$G_1=\frac{(1+r)\tau_CY_1+\tau_CY_2^{UN}(\tau_C)-(1+r)Z_1^{UN}(\tau_C)-Z_2^{UN}(\tau_C)}{1+r+\{(1+r)\delta\}^{\frac{1}{\zeta}}} \tag{16-6}$$

Substituting (16-6) into (16-5), we obtain a solution of τ_C, denoted by τ_C^{UN}. Also G_1 and G_2 are decided as follows

$$G_1^{UN}\equiv\frac{(1+r)\tau_C^{UN}Y_1+\tau_C^{UN}Y_2^{UN}(\tau_C^{UN})-(1+r)Z_1^{UN}(\tau_C^{UN})-Z_2^{UN}(\tau_C^{UN})}{1+r+\{(1+r)\delta\}^{\frac{1}{\zeta}}}$$

and

$$G_2^{UN}\equiv\{\delta(1+r)\}^{1/\zeta}G_1^{UN}.$$

Therefore, all policy variables are determined as $\tau_C^{UN},G_1^{UN},G_2^{UN}$, $g_1^{UN}(\tau_C^{UN}),g_2^{UN}(\tau_C^{UN})$, $k^{UN}(\tau_C^{UN})$, and $\tau_L^{UN}(\tau_C^{UN})$ In this situation, government debts of the central and local governments are determined as follows

$$B^{UN}=G_1^{UN}+Z_1^{UN}(\tau_C^{UN})-\tau_C^{UN}Y_1$$

$$D^{UN} = g_1^{UN}(\tau_C^{UN}) + k^{UN}(\tau_C^{UN}) - Z_1^{UN}(\tau_C^{UN}) - Y_1\tau_L^{UN}(\tau_C^{UN})$$

4.5 Situation in Federal States with Bond Issuance Cap

In situation in unitary states with bond issuance cap, as described in Section 3.4, the optimality conditions of both CG and LG are the following

$$\left(\frac{g_1}{g_2}\right)^{-\theta} = \frac{1-m_1}{1-m_2}\frac{\tau_L\delta A\alpha k^{\alpha-1}}{1-m_k} \quad (17\text{-}1)$$

$$2\Omega\omega_L\tau_L(\omega_C\tau_C + \omega_L\tau_L) = \left(1+\frac{\tau_L A\alpha k^{\alpha-1}}{1-m_k}\right)Y_1 + Ak^\alpha - \Omega(\omega_C\tau_C + \omega_L\tau_L)^2 \quad (17\text{-}2)$$

Substituting (3-1') and (3-2') into (17-1), we obtain

$$\left(\frac{1-m_2}{1-m_1}\frac{\bar{D}-(1-m_k)k-\tau_L Y_1}{\tau_L Y_2-(1+r)\bar{D}}\right)^{-\theta} = \frac{1-m_1}{1-m_2}\frac{\tau_L\delta A\alpha k^{\alpha-1}}{1-m_k} \quad (17\text{-}1')$$

On the other hand, the optimality conditions of CG are as follows

$$\left(\frac{G_1}{G_2}\right)^{-\zeta} = (1+r)\delta \quad (17\text{-}4)$$

$$2\Omega\omega_C\tau_C(\omega_C\tau_C + \omega_L\tau_L) = (2+r)Y_1 + Ak^\alpha - \Omega(\omega_C\tau_C + \omega_L\tau_L)^2 \quad (17\text{-}5)$$

In this Nash equilibrium, we have

$$(\omega_C\tau_C)^2 = (\omega_L\tau_L)^2 + \frac{Y_1}{2\Omega}\left(1+r-\frac{\tau_L A\alpha k^{\alpha-1}}{1-m_k}\right) \quad (17\text{-}6)$$

from (17-3) and (17-5). Therefore, we obtain

$$\tau_C = \frac{1}{\omega_C}\sqrt{(\omega_L\tau_L)^2 + \frac{Y_1}{2\Omega}\left(1+r-\frac{\tau_L A\alpha k^{\alpha-1}}{1-m_k}\right)} \quad (17\text{-}6')$$

since τ_C is non negative.

(17-5) is rewritten as

$$2\Omega\omega_C\tau_C(\omega_C\tau_C + \omega_L\tau_L) + \Omega(\omega_C\tau_C + \omega_L\tau_L)^2 = (2+r)Y_1 + Ak^\alpha$$

$$\Omega\{3(\omega_C\tau_C)^2 + 4(\omega_C\tau_C)(\omega_L\tau_L) + (\omega_L\tau_L)^2\} = (2+r)Y_1 + Ak^\alpha \quad (17\text{-}5')$$

Substituting (17-6) and (17-6') into (17-5'), we have

$$4\Omega(\omega_L\tau_L)^2 + \frac{3Y_1}{2}\left(1+r-\frac{\tau_L A\alpha k^{\alpha-1}}{1-m_k}\right) + 4\Omega\omega_L\tau_L\sqrt{(\omega_L\tau_L)^2 + \frac{Y_1}{2\Omega}\left(1+r-\frac{\tau_L A\alpha k^{\alpha-1}}{1-m_k}\right)}$$
$$= (2+r)Y_1 + Ak^\alpha$$

$$4\Omega(\omega_L\tau_L)^2+4\Omega\omega_L\tau_L\sqrt{(\omega_L\tau_L)^2+\frac{Y_1}{2\Omega}\left(1+r-\frac{\tau_L A\alpha k^{\alpha-1}}{1-m_k}\right)} \\ =\frac{Y_1}{2}\left(1+\frac{3\tau_L A\alpha k^{\alpha-1}}{1-m_k}-r\right)+Ak^{\alpha} \quad (17\text{-}7)$$

Therefore, solving system of equations, (17-1') and (17-7), we find a solution of τ_L and one of k, denoted by $\tau_L^{FC}\big|_{D=\bar{D}}$ and $k^{FC}\big|_{D=\bar{D}}$, respectively. Solutions of τ_L and k depend on $\bar{D}$, but it is not a variable in this setting.

In this Nash equilibrium, from the above conditions, we have

$$\tau_C^{FC}\equiv\frac{1}{\omega_C}\sqrt{(\omega_L\tau_L^{FC})^2+\frac{Y_1}{2\Omega}\left(1+r-\frac{\tau_L A\alpha(k^{FC})^{\alpha-1}}{1-m_k}\right)}$$

In this case compared with the case without bond issuance cap in federal states, when $r>\frac{\tau_L A\alpha(k^{FC})^{\alpha-1}}{1-m_k}$, $>\frac{\omega_L}{\omega_C}\tau_L$

**** under preparation ****

Also we obtain the following by using (17-6')

$$Y_2^{FC}\equiv Y_1+A(k^{FC})^{\alpha}-\Omega\left\{\omega_L\tau_L^{FC}+\sqrt{(\omega_L\tau_L^{FC})^2+\frac{Y_1}{2\Omega}\left(1+r-\frac{\tau_L^{FC}A\alpha(k^{FC})^{\alpha-1}}{1-m_k}\right)}\right\}^2$$

Thus

$$g_1^{FC}\equiv\frac{1}{1-m_1}\{\bar{D}-(1-m_k)k^{FC}+\tau_L^{FC}Y_1\}$$

$$g_2^{FC}\equiv\frac{1}{1-m_2}\{\tau_L^{FC}Y_2^{FC}-(1+r)\bar{D}\}$$

and

$$Z_1^{FC}\equiv m_1 g_1^{FC}+m_k k^{FC}$$

$$Z_2^{FC}\equiv m_2 g_2^{FC}$$

From the above solutions, we have

$$G_1^{FC}\equiv\frac{(1+r)\tau_C^{FC}Y_1+\tau_C^{FC}Y_2^{FC}-(1+r)Z_1^{FC}-Z_2^{FC}}{1+r+\{(1+r)\delta\}^{\frac{1}{\zeta}}}$$

and

$$G_2^{FC}\equiv\{\delta(1+r)\}^{1/\zeta}G_1^{FC}.$$

from (17-4).

In this Nash equilibrium, government debts of the central and local governments are determined as follows

$$B^{FC} = G_1^{FC} + Z_1^{FC} - \tau_C^{FC} Y_1$$

4.6 Situation in Unitary States with Bond Issuance Cap

In situation in unitary states with bond issuance cap, as described in Section 3.5, the optimality conditions of both CG and LG are the following

$$\left(\frac{g_1}{g_2}\right)^{-\theta} = \frac{1-m_1}{1-m_2}\frac{\tau_L \delta A \alpha k^{\alpha-1}}{1-m_k} \quad (18\text{-}1)$$

$$2\Omega\omega_L \tau_L (\omega_C \tau_C + \omega_L \tau_L) = \left(1 + \frac{\tau_L A \alpha k^{\alpha-1}}{1-m_k}\right) Y_1 + Ak^{\alpha} - \Omega(\omega_C \tau_C + \omega_L \tau_L)^2 \quad (18\text{-}2)$$

(18-1) and (18-2) are the same as (17-1) and (17-2). Substituting (3-1') and (3-2') into (18-1), we obtain (17-1'). Therefore, solving system of equations, (17-1') and (18-2), we find a response function with respect to τ_L and one of k, denotes $\tau_L^{UC}(\tau_C;\bar{D})$ and $k^{UC}(\tau_C;\bar{D})$, respectively. Then, we have these response functions from the above conditions;

$$g_1^{UC}(\tau_C) \equiv \frac{1}{1-m_1}\{\bar{D} - (1-m_k)k^{UC}(\tau_C) + Y_1 \tau_L^{UC}(\tau_C)\}$$

$$g_2^{UC}(\tau_C) \equiv \frac{1}{1-m_2}\{\tau_L^{UC}(\tau_C) Y_2^{UC}(\tau_C) - (1+r)\bar{D}\}$$

where $Y_2^{UC}(\tau_C) = Y_1 + A\{k^{UC}(\tau_C)\}^{\alpha} - \Omega\{\omega_C \tau_C + \omega_L \tau_L^{UC}(\tau_C)\}^2$

and

$$Z_1^{UC}(\tau_C) \equiv m_1 g_1^{UC}(\tau_C) + m_k k^{UC}(\tau_C)$$

$$Z_2^{UC}(\tau_C) \equiv m_2 g_2^{UC}(\tau_C)$$

On the other hand, the optimality conditions of CG are as follows

$$\begin{aligned}
&\left\{\frac{\partial g_1^{UC}}{\partial \tau_C} + \frac{(1-m_2)(1-m_k)}{(1-m_1)\delta A \alpha \{k^{UC}(\tau_C)\}^{\alpha-1} \tau_L^{UC}(\tau_C)} \frac{\partial g_2^{UC}}{\partial \tau_C}\right\}\{g_1^{UC}(\tau_C)\}^{-\theta} \\
&= G_1^{-\zeta}\left[\left\{m_1 \frac{\partial g_1^{UC}}{\partial \tau_C} + m_k \frac{\partial k^{UC}}{\partial \tau_C} + \frac{m_2}{1+r}\frac{\partial g_2^{UC}}{\partial \tau_C}\right\}\right. \\
&\quad - \left\{Y_1 + \frac{Y_2^{UC}(\tau_C)}{1+r} + \frac{\tau_C}{1+r}\left(A\alpha (k^{UC})^{\alpha-1}\frac{\partial k^{UC}}{\partial \tau_C}\right.\right. \\
&\qquad \left.\left.\left. -2\Omega\{\omega_C \tau_C + \omega_L \tau_L^{UC}(\tau_C)\}\{\omega_C + \omega_L \frac{\partial \tau_L^{UC}}{\partial \tau_C}\}\right)\right\}\right]
\end{aligned} \quad (18\text{-}3)$$

$$\left(\frac{G_1}{G_2}\right)^{-\zeta} = (1+r)\delta \tag{18-4}$$

From (2-3') and (18-3), we have

$$G_1 = \frac{(1+r)\tau_C Y_1 + \tau_C Y_2^{UC}(\tau_C) - (1+r)Z_1^{UC}(\tau_C) - Z_2^{UC}(\tau_C)}{1+r+\{(1+r)\delta\}^{\frac{1}{\zeta}}} \tag{18-5}$$

Substituting (18-5) into (18-3), we obtain a solution of τ_C, denoted by τ_C^{UC}. Also G_1 and G_2 are decided as follows

$$G_1^{UC} \equiv \frac{(1+r)\tau_C^{UC} Y_1 + \tau_C^{UC} Y_2^{UC}(\tau_C^{UC}) - (1+r)Z_1^{UC}(\tau_C^{UC}) - Z_2^{UC}(\tau_C^{UC})}{1+r+\{(1+r)\delta\}^{\frac{1}{\zeta}}}$$

and

$$G_2^{UC} \equiv \{\delta(1+r)\}^{1/\zeta} G_1^{UC}.$$

Therefore, all policy variables are determined as $\tau_C^{UC}, G_1^{UC}, G_2^{UC}$, $g_1^{UC}(\tau_C^{UC}), g_2^{UC}(\tau_C^{UC})$, $k^{UC}(\tau_C^{UC})$, and $\tau_L^{UC}(\tau_C^{UC})$ In this situation, government debts of the central and local governments are determined as follows

$$B^{UC} = G_1^{UC} + Z_1^{UC}(\tau_C^{UC}) - \tau_C^{UC} Y_1$$

4.7 Data and Parameters

In this section, we prepare to implement numerical analyses based on our model described above. Parameters in our model are set as shown in Table 1. δ is assumed to be 0.8179, which is approximately equal to the twentieth power of 0.99. Interest rate is set at 3%. Parameters of productivity of public investment, A and α, are adjusted in order to get plausible results in this numerical analyses. Also parameters of preference for public goods, θ and ζ, are adjusted for similar reasons. For simplicity, grant rates for local expenditures, m_1 and m_2 are set at 20%, m_k is set at 40%, and bond issuance cap is set at 10% of Y_1.

4.8 Results

Results in our numerical analyses are shown in Table 2. We present the result of the case in federal states without bond issuance cap, based on equations in section 4.3, in Row DN. W denotes the social welfare calculated by (11). In our setting, the social welfare in this case is highest, though it depends on values of parameters.

We also show the result of the case in unitary states without bond issuance cap, based on equations in section 4.4, in Row UN. The results suggest that the social welfare in this case is lower than that in the case of DN. The reason may be excess burden with taxation due to overborrowing of governments.

Results of the case in federal states with bond issuance cap, based on equations in section 4.5, are described in Row DC. Bond issuance cap is set at 20% of Y_1. As indicated in Table 2, the social welfare in this case is lower than that in the case of DN. In the case of DN, the local government chooses fewer bond issuance based on optimality condition than the bond issuance cap. In this setting, bond issuance cap prompts borrowing of the local government, though it is inefficient.

Finally, we also demonstrate the result of the case in unitary states with bond issuance cap, based on equations in section 4.6 in Row UC. The results suggest that the social welfare in this case is higher than that in the case of UN. In this case, bond issuance cap restricts borrowing of the local government.

5. Concluding Remarks

In this paper, we have investigated theoretically and numerically allocation of government debt between the central and local governments by clarifying the vertical externality of local expenditures due to overlapping tax bases between two governments using a two-period model.

At the present stage, our investigation is very limited. In results of this paper, we just set one numerical example. We need further research based on our model.

Appendix: Multiple local governments

Suppose there are n local governments. If we define the total amount of local public goods as g_1, g_2 and each local government's supply of public goods as g_1^i, g_2^i, then we have

$$g_1 = \sum_{i=1}^{n} g_1^i\,, \quad g_2 = \sum_{i=1}^{n} g_2^i \tag{A1}$$

The social welfare (1) is now rewritten as

$$W^i = u(G_1) + v(g_1^i) + \delta\{u(G_2) + v(g_2^i)\} \tag{A2}$$

where W^i is the social welfare in the representative agent in region i.

$$W = \sum_{i=1}^{n} W^i$$

We may define other variables of local governments as in (A1). Then the budget constraints of CG and LG are the same as in the text. For simplicity suppose all local governments are identical. It follows that in the section of 2.2 the first best conditions are given by

$$v_{g1} = nu_{G1} \tag{A3-1}$$

$$u_{G2} = nv_{g2} \tag{A3-2}$$

and (5-3), (5-4). (A3-1) and (A3-2) correspond to the well-known Samuelson condition of the pure public good, G. We have analytically the same results as in section 2.2.

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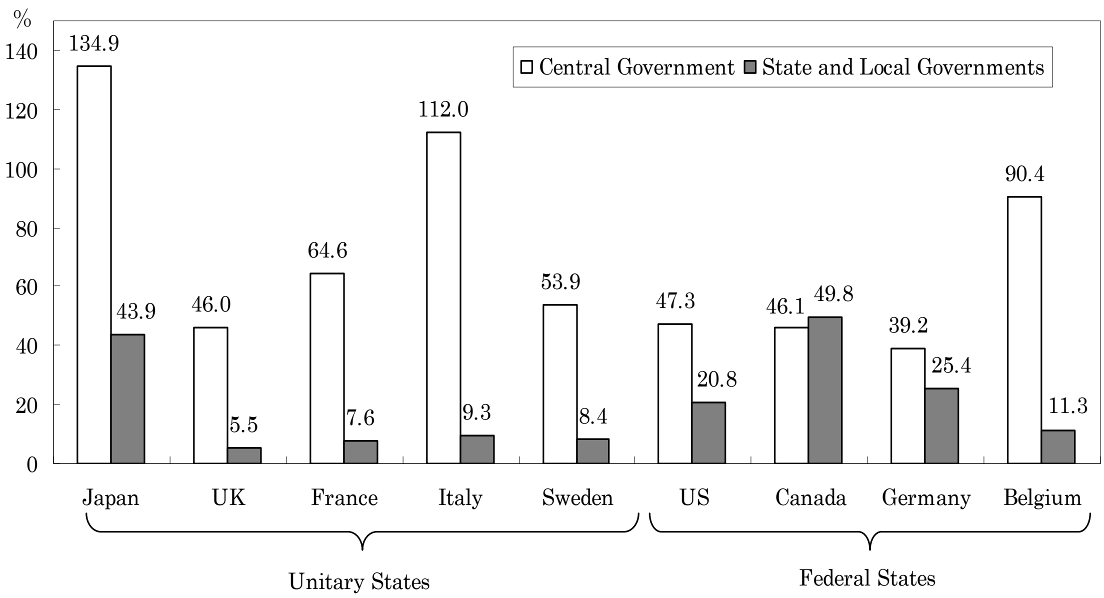
Figure 1

Government Debt to GDP Ratio in 2004



Source: OECD National Accounts

Table 1
Parameter Values

r	0.03
A	10
α	0.8
Y_1	10000
δ	0.8179
θ	0.6
ζ	0.6667
Ω	19000
ω_C	1
ω_L	1.01
m_1	0.2
m_2	0.2
m_k	0.4
$\bar{D}$	1000

Table 2

Results of Numerical Analyses

	DN	UN	DC	UC
W	139.08	133.89	136.58	138.13
g_1	1940.59	1152.66	1407.07	1628.24
G_1	1444.52	2470.72	2137.98	1850.56
g_2	1481.94	880.23	1074.51	1243.41
G_2	1133.26	1938.33	1677.30	1451.81
D/Y_1	0.125	0.326	0.200	0.200
B/Y_1	0.096	0.127	0.108	0.032