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Optimal Commodity Taxation in the Presence of Involuntary Unemployment:[1]
Importance of Progressive Income Taxation and Unemployment Insurance

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Abstract: This paper addresses the optimal commodity taxation when involuntary unemployment arises from redistributive fiscal policies, e.g., a progressive income tax system and social insurance/welfare programs. This setting deals with a new dimension of distortions: the "between-states" (employment state vs. unemployment state) consumption choice distortion along with the usual "within-state" distortion conditional on a given state. We derive the optimal commodity taxation rule in the presence of the redistributive fiscal policies in a simplified 'general-equilibrium' efficiency wage model with effort and commodity choices. In contrast to the conventional results by Ramsey (1927) and Atkinson and Stiglitz (1976) that consider the within-state distortion only, we show that under the weakly separable utility and the constant marginal cost technology, uniform commodity taxation is optimal only when the government can choose *all* commodities' tax rates. If at least one good's tax rate is fixed at a certain level (e.g., due to redistributive purpose or to foreign competition), non-uniform commodity taxation is generally optimal.

The intuition is that a deviation from uniform commodity taxation can alleviate the between-states distortion, moral hazard arising from a progressive income tax system and social insurance/welfare programs. This gain, combined with resulting greater effort and higher utilization of labor (i.e., lower unemployment), can outweigh the within-state consumption choice distortions from non-uniform taxation. Useful policy implications are also discussed.

Key Words: between-states vs. within-state distortions, involuntary unemployment, optimal taxation, progressive income taxation, efficiency wages, moral hazard, utilization of labor, strategic complementarity, normalization.

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I. Introduction

One of the central policy issues in modern economies can be said unemployment. While the nature of unemployment is diverse,[2] unemployment in many developed countries stems in part from redistributive fiscal policies. For instance, the nature of unemployment in some welfare states may be viewed in part as a combination of (i) a low work incentive induced by progressive income tax and redistributive welfare systems and (ii) the subsequent "dismissals" (e.g., not renewing employment contracts or selecting part of temporary/part-time workers as regular workers) by firms to raise labor productivity. To deal with this type of unemployment, the government may seek policies such as alleviating income tax progressivity or lowering unemployment benefits, but redistributive motives by the public often do not permit implementation of those policies. Under these socioeconomic constraints, this paper asks a question: Can commodity taxation help reduce unemployment and improve efficiency?

Since Ramsey (1927), optimal commodity taxation has been studied under the traditional perfect information assumption or under imperfect information about type of individuals (Atkinson and Stiglitz (1976)). However, to our knowledge, there is little research exploring the role of commodity taxation in the presence of involuntary unemployment.[3] To deal with the role of commodity taxes in the presence of the involuntary unemployment described above, this paper develops a simplified general equilibrium efficiency wage model with imperfect information about a worker's effort supply. Then we derive some useful results on the optimal commodity tax rule. In contrast to the conventional wisdom, we show that uniform commodity

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[2] See Johnson and Layard (1986) for a survey of modern unemployment and the policies.

[3] Most studies in the literature considered commodity taxation in the traditional general equilibrium context. Some studies with imperfect information looked at the situation where an individual's type is not observable (Atkinson and Stiglitz (1976)). Marchand et al. (1989) considered the optimal commodity taxation in a situation where fixed real wages caused excess supply of labor, a typical Keynesian unemployment. Their intuitive graphical arguments are certainly appealing, but we consider a different version of "equilibrium unemployment" that stems from information imperfection. To our knowledge, there is little research examining the role of commodity taxation in this type of environment.

taxation is optimal only when the government can choose *all* commodities' tax rates without any constraint. In a more realistic case where there is at least one constraint, non-uniform commodity taxation is optimal even under (i) the utility function that is weakly separable between goods and leisure and (ii) the constant marginal cost technology.

From the Ramsey taxation to more modern optimal taxation analyses, uniform commodity taxes have been known as optimal under the functional assumptions of weakly separable utility and constant marginal cost production. For example, under those assumptions, Ramsey's main results suggest that uniform taxes are optimal. In a more general model with heterogeneous agents and a redistributive motive of a government, using the seminal work of Mirrlees (1971), Atkinson and Stiglitz (1976) showed that optimal commodity taxes are uniform if (i) the utility function is weakly separable between commodities and leisure, and (ii) the marginal cost of production is constant. This "uniform commodity taxation" result has been treated as one of the classic findings in the optimal taxation literature.[4] Combined with the result of "production efficiency" by Diamond and Mirrlees (1971), the result of uniform commodity taxation suggests that (i) even if the government is concerned with income redistribution, it should not distort the relative price structure of the commodity markets, and (ii) income redistribution can be sufficiently dealt with by non-linear (progressive) income taxation only.

This paper examines the commodity tax problem in a different context of information imperfection: it is difficult for firms to perfectly observe the effort supply of their workers. Considering the presence of progressive income tax and UI systems, the imperfect information assumption can add more realism to the existing setting that is not able to deal with involuntary

[4] Different conclusions are possible under different assumptions. In a similar setting with Atkinson and Stiglitz but with a different assumption that the marginal cost of production is *not* constant, Naito (1999) showed that under the weakly separable utility function, non-uniform commodity taxes can be optimal. The main intuition is that doing so would relax the incentive-compatibility condition and boost the labor supply of unskilled workers.

unemployment and UI. Perhaps, the underlying environment of our model may be more consistent with many developed countries' concerns about high unemployment arising from generous UI and progressive income taxes. We show that welfare improvement becomes possible by non-uniform commodity taxes under the weakly separable utility and the constant marginal cost technology.

To illustrate the intuition behind our result, consider a situation where (i) income tax progressivity is determined by the redistributive motives of individuals, (ii) the government sets the levels of income and commodity tax rates so as to finance the expenditures for UI and public goods, and (iii) the existing commodity taxes are uniform. Given the difficulty of observing effort, a progressive income tax system combined with redistributive social insurance is equivalent to treating the consumption of unemployed workers favorably (i.e., "between-states" consumption choice distortion), which thus creates moral hazard in effort supply. Introduction of non-uniform commodity taxation can lower this "between-states" consumption choice distortion, leading to a greater effort supply. The increased effort means a lower equilibrium unemployment, and it leads to a greater *utilization* of labor and hence to a greater output. These gains can outweigh the "within-state" consumption choice distortion arising from the non-uniform taxation.

Our model structure also warrants some discussion. We develop a new shirking-type efficiency wage framework for analyzing the welfare effects of fiscal policy changes (e.g., changes in income and commodity taxes and the UI system) in the presence of imperfect information on effort. A useful feature of our model is that unlike previous studies, it permits a complete general equilibrium analysis such that policy effects interact between the labor and commodities markets.[5] [6] We also incorporate the institutions of UI and the progressive income

[5] For the traditional structure of the efficiency wage models of unemployment, see Johnson and Layard (1986) and Yellen (1984) along with Shapiro and Stiglitz (1984). As for more recent studies, Pisauro (1991) and Agell and Lundborg (1992) applied the traditional efficiency wage models to taxation analysis, but disequilibrium features are

tax system into our optimal taxation analysis to deal with the interaction between work and unemployment.

This paper is organized as follows. The next section describes our model and presents its key features. Section III presents the optimal tax problem under perfect and imperfect information, respectively, and shows the properties of the useful results. Perhaps a more interesting policy issue of the commodity tax effect on unemployment is analyzed in Section IV. Some policy implications are derived to shed light on (i) the tax policies of welfare states and (ii) the recently emerging issue of the double dividend hypothesis regarding environmental tax reform as well. The final section summarizes the key results of the paper.

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built in these models.

[6] The literature on eliciting worker effort under information imperfection is extensive. The motivation for this line of studies is based on the view that the labor market is *not* perfect as most papers in traditional general equilibrium models assume. Since Shapiro and Stiglitz (1984), recent decades have seen a rapid development of the efficiency wage theory and its applications to various fields of economics. Despite many useful properties (e.g., explaining involuntary unemployment), however, the efficiency wage models focus on the labor market equilibrium excluding other markets (e.g., the goods market), and thus we usually obtain 'partial' equilibrium or 'disequilibrium' in nature. For example, the formal sector offers efficiency wages while the informal sector pays the competitive wage that is *fixed* at a much lower level (e.g., Shapiro and Stiglitz (1984)). In a shirking-version efficiency wage model like Johnson and Layard (1986), firms set efficiency wages so as to maximize profits, but the profits are not endogenized in the form of incomes to be spent on commodities, so a complete general equilibrium analysis is not possible. Hence, it may not be surprising that welfare effects of policy changes are generally not addressed in the efficiency wage framework, not to mention the optimal tax rule. In contrast, our model can permit welfare analyses while holding the essence of efficiency wage models.

II. The Model

1. Nature of unemployment and institutional environment

Following the tradition of efficiency wage theory (e.g., Shapiro and Stiglitz (1984)), we assume that there are *N* identical workers with time endowment $\overline{T}$ (e.g., $\overline{T}$ = 24 hours per day) to be assigned to two states: employment (state 1) or unemployment (state 2). For state 1, workers should stay in their firms for a required amount of time *L* in accordance with employment contract (e.g., nine to five o'clock per day), but they can choose their own effort level while taking the risk of being fired when they are caught in shirking. Thus in our model, workers can adjust effort supply *e* (or effective labor supply). If they shirk for a fraction $1-e$ (e is defined to be $0 \le e \le 1$) of their working time *L*, they face the risk of being fired with probability $\pi(e,d)$, where *d* is the probability that a representative firm monitors workers.[7] In this case the true working time becomes *eL* and the amount of shirking time is (1-*e*)*L* (henceforth *L* will be normalized to be unity). Knowing the structure of the probability of being fired, individuals choose their optimal effort and consumption of two goods (X_1 and X_2) as well.[8] When caught in shirking, workers are fired and receive unemployment benefits. In this sense, individuals are *ex ante* identical but ex post heterogeneous (i.e., workers are either employed or unemployed), and the *ex post* worker types are revealed by their employment status. In our setting, the labor supply decision is dichotomous, either work (employed) or no work (unemployed), while the effort decision is continuous.

Given the two types of individuals in the economy, redistributive motives of

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[7] For example, if a firm monitors all the workers, then *d* will be equal to one, and if it does only a selected sample, then *d* will be less than one. Because monitoring is costly, the firm will choose *d* optimally according to the marginal cost of monitoring.

[8] Of course, our model can be generalized to the case of n goods, but for the sake of brevity, we focus on the two goods case.

unemployed workers lead to a progressive income tax system that applies a lower tax rate to unemployed workers' incomes ($b<w$): the tax rates on labor incomes and unemployment benefits are t_w and $t_b = \phi t_w$, respectively, where ϕ is the parameter for the income-tax progressivity, and $0 \le \phi < 1$. ϕ can be treated as a choice variable of the government, but without loss of generality, we assume that the size of ϕ is determined by political process while the government sets the level of t_w so as to finance the expenditures on public goods and UI (i.e., balanced budget constraint). The progressivity in our model is dichotomous: a high tax rate t_w for working individuals and a low rate t_b for unemployed workers. Although this is a highly simplistic treatment of income tax progressivity, it is general enough to capture the main intuition of this paper. In addition, the progressivity given here plays not only a redistributive role but also an income-insurance role in the sense of Varian (1980).[9] Another institution is the UI system, which provides unemployment benefit b to unemployed workers. The UI expenditure is financed by income and commodity taxes.[10]

When these institutions are combined with some sort of imperfect information about individuals' effort level, some extent of moral hazard is inevitable. Individuals decide their optimal level of effort, knowing the structure of UI and the redistributive extent of the tax policies about income and consumption in terms of work incentives. Other things being constant, as the redistributive nature of the income tax system and UI becomes stronger, the greater will be the *negative* work incentives.[11] This labor market distortion considered in this paper is therefore

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[9] Varian (1980) showed that if income is subject to randomness, then income tax can play the role of social insurance. In our case, workers are "randomly" monitored and laid off with some probability, so insurance for unemployment by shifting income from the employment state to the unemployment state may be Pareto-improving to some extent.

[10] An implicit assumption is that the level of UI benefits is set such that the utility under employment is higher than that under unemployment, an incentive compatibility condition. Since the level of b is not a primary issue here, we do not explicitly consider this condition when solving the model.

[11] Of course, UI can be a useful welfare-improving insurance program for unexpected job losses, but its policy goals are often contaminated by distributional concern as found in many welfare states. Empirically, it seems that countries with high unemployment rates do not necessarily face a greater extent of random shocks than other countries with

slightly different from that in traditional models: while the traditional studies look at the distortions in labor supply caused by income taxation under the perfect labor market assumption, we consider the distortions under the imperfect labor market assumption.

2. Consumer's Decisions

A representative individual's utility function is given by $W\left(U(Q(X_1, X_2), \bar{T} - e), G\right)$, where private goods are weakly separable from public good *G*, and commodities and leisure 1-*e* are also weakly separable. Given commodities X_1 and X_2 are weakly separable from $\bar{T} - e$ (leisure), two-stage budgeting solutions are possible as follows.

Two-Stage Budgeting Solution

Given that public good *G* is not a choice variable for individuals and there exists separability between private and public goods, we now focus on the sub-utility function *M* rather than *U*. The probability that a worker belongs to the state of employment (state 1) is 1- (1- *e*)*d* and the counterpart for the state of unemployment (state 2) is (1- *e*)*d*. In state 1, the worker's problem is to make consumption choices with net income (1-t_w)*w* while in state 2, that is with net unemployment compensation $(1-\phi t_w)\, b$. Since the utility function *M* is weakly separable, we can solve the problem using the two-stage budgeting technique. At the first stage, the worker determines the level of effort. The employment status is also determined conditional on firms' monitoring. The second stage determines consumption of X_1 and X_2 based on employment status. Given the sequential nature of our problem, we will begin with the second-stage problem.

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low unemployment rates. To focus on the unemployment caused by the fiscal policies, i.e., progressive income tax and social welfare, we abstract from any exogenous random shocks and assume that the state a worker belongs to is affected solely by the effort level that he or she *chooses*.

(1) second-stage problem

State 1: $$\max_{X_1^{em}, X_2^{em}} U\left(Q(X_1^{em}, X_2^{em}), \overline{T} - e\right) \qquad (1)$$

$$\text{s.t.} \quad (1+t_1)X_1^{em} + (1+t_2)X_2^{em} = (1-t_w)w,$$

where superscript *em* stands for employment (e.g., X_1^{em} is the consumption of X_1 for the state of employment), t_i is the consumption tax rate on commodity *i* (*i* =1, 2), t_w is the income tax rate and *w* is the wage (the total compensation paid by the firm). Similarly, the state 2 problem is:

State 2: $$\max_{X_1^{un}, X_2^{un}} U\left(Q(X_1^{un}, X_2^{un}), \overline{T} - 1\right) \qquad (2)$$

$$\text{s.t.} \quad (1+t_1)X_1^{un} + (1+t_2)X_2^{un} = (1-\phi t_w)b,$$

where superscript *un* stands for unemployment (e.g., X_1^{un} is the consumption of X_1 for the state of unemployment).

(2) first-stage problem

Once the state-contingent commodity demand functions are obtained from the second stage, effort supply is determined by solving the following first-stage problem subject to the given Marshallian demands for state-contingent goods:

$$\max_{e} \ (1-\pi(e,d))U\left(Q(X_1^{em}, X_2^{em}), \overline{T} - e\right) + \pi(e,d)U\left(Q(X_1^{un}, X_2^{un}), \overline{T} - 1\right) \qquad (3)$$

subject to $$X_1^{em} = X_1^{em}(1+t_1, 1+t_2, (1-t_w)w),$$

$$X_1^{un} = X_1^{un}(1+t_1, 1+t_2, (1-\phi t_w)b),$$

$$X_2^{em} = X_2^{em}(1+t_1, 1+t_2, (1-t_w)w),$$

$$X_2^{un} = X_2^{un}(1+t_1, 1+t_2, (1-\phi t_w)b) ,:$$

In this problem, the effort level is chosen after considering not only the utility from leisure but also the chance of unemployment and fiscal policy parameters. The first-order condition with respect to *e* is as follows:[12]

$$\pi_e (U^{em} - U^{un}) + (1 - \pi(e,d)) U^{em}_{\bar{T}\text{-}e} = 0 , \qquad (4)$$

where $\pi_e = \partial \pi(\cdot) / \partial e < 0$ and $U^{em}_{\bar{T}\text{-}e} = \partial U^{em}(\cdot) / \partial(\bar{T} - e) > 0$.

Although the first-order condition does not give a closed-form solution for effort, we can verify the following properties for *e*: (i) an increase in wage boosts effort, that is, $\partial e / \partial w > 0$ for $e = e(t_1, t_2, t_w, \phi, w, b, d, \alpha_3)$; similarly, an increase in income tax rate will lead to a decrease in effort, that is, $\partial e / \partial t_w < 0$; (ii) a more progressive income tax system leads to a lower effort, $\partial e / \partial \phi > 0$; (iii) a more generous UI system reduces effort, $\partial e / \partial b < 0$; (iv) a greater detection rate elicits more effort, $\partial e / \partial d > 0$; (v) $\partial e / \partial t_i \geq 0$; since it is usually said that commodity taxation is regressive or more painful for low-income unemployed people in terms of utility, we assume that increases in the commodity tax rates lead to an increase in effort. In fact, all these properties hold under the CES utility function.

3. *Producer's Decisions*

Following the standard assumption of the constant marginal cost technology used in most studies on optimal taxation, we adopt a simplest possible form among the category of the constant marginal cost technologies: output is produced with a single production factor, effort, and hence the production function is *f*(*e*) = *e*. It can be used for the production of either public

[12] The utility function is concave with respect to effort: $-\pi_{ee}(U^{em} - U^{un}) + 2\pi_e U^{em}_{\bar{T}\text{-}e} + (1-\pi) U^{em}_{\bar{T}\text{-}e,\bar{T}\text{-}e} < 0$, where $\pi_e < 0$ and $\pi_{ee} \geq 0$ are used (and the signs also make sense: a greater effort leads to a lower separation from a job and a further effort has a smaller impact on reducing the separation rate). There exists a unique maximum for *e*. Since we need to address non-zero unemployment, focusing on the interior solution is justified.

good, G or private goods, X_1 and X_2. Accordingly, equilibrium in the commodity market is given by:

$$\begin{aligned} eN(1-\pi(e,d)) &= N(X_1 + X_2) + G \\ &= N[(1-\pi(e,d))(X_1^{em} + X_2^{em}) + \pi(e,d)(X_1^{un} + X_2^{un})] + G, \end{aligned} \tag{5}$$

where units are normalized such that the rates of transformation among three goods, X_1, X_2 and G, are unity. A key feature of our model is that only the effort exerted from the employed workers $eN(1-\pi(e,d))$ leads to production since other workers will be fired. Note that since an increase in effort gives rise to an increase in the rate of "utilization of labor" (i.e., $\partial(1-\pi(e,d))/\partial e > 0$), effort and utilization of labor are strategic complementarities, so we have a "super-modularity" result here. Although the production function exhibits constant-returns-to-scale, the actual production takes place at *increasing returns to scale* (IRS) with respect to effort. This production possibility frontier is derived from solving the problem of firm *i* with n_i workers:

$$\max_{w,d}(1-\pi(e,d))(e-w-cd)n_i \qquad \text{s.t. } e = e(t_1, t_2, t_w, \phi, w, b, d, \alpha_3)\,. \tag{6}$$[13]

Profit maximization leads to the following first-order condition: [14]

$$\pi_e(e-w-cd) = (1-\pi)(e_w - 1) \tag{7}$$

$$\partial e(\cdot)/\partial d = c\,. \tag{8}$$

Then, the perfect competition assumption leads to a zero profit condition, $e = w\text{-}cd$, and it is straightforward to obtain the condition e_w =1 from equation (7). Note that e_w =1 suggests that marginal cost is equal to marginal revenue.

4. Government's Decisions

The last equation for our model is about the government budget constraint. The

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[13] The detection rate *d* is assumed to be exogenous. In the basic model with a Cobb-Douglas utility, it can be easily shown that endogenizing *d* does not affect the result, so this is not a restrictive assumption.

[14] The second order condition was checked; uniqueness of the solution is satisfied in our model if e_{ww} < 0.

government can collect taxes from labor income and commodity consumption to finance the expenditure on public good G and the unemployment benefits paid to jobless workers:

$$N\left[(1-\pi(e,d))\left(t_w w+\sum_{i=1}^{2} t_i X_i^{em}\right)+\pi(e,d)\left(\phi t_w b+\sum_{i=1}^{2} t_i X_i^{un}\right)\right]=\overline{G}+N\pi(e,d)b \qquad (9)$$

5. General Equilibrium

Given the standard quasi-concave utility and convex production possibility frontier, the existence of equilibrium can be shown straightforwardly. The general equilibrium with three endogenous variables $\{X_1, X_2, e\}$ can be characterized by the following equations describing (i) the effort function (equation (4)), (ii) the production possibility frontier (equation (5)), (iii) the balanced budget condition (equation (8)) and (iv) zero profit condition ($w = e$) combined with $e_w = 1$. According to Walras' law, one of the equations is redundant with the other equations, so any three equations among the equations above can be used to analyze the general equilibrium.[15]

[15] For instance, the government budget constraint can be derived from substituting individuals' state-contingent budget constraints and market-clearing conditions into the production possibility frontier.

III. Optimal Commodity Taxation

1. Structure of optimal commodity tax under perfect information

If information is perfect, then costless perfect monitoring can be carried out and there will be no unemployment. In this case, the consumer problem is simply maximizing the State 1 utility only:

$$\max_{X_1^{em}, X_2^{em}} U\left(Q(X_1^{em}, X_2^{em}), \overline{T} - e\right) \tag{10}$$

s.t. (i) $(1+t_1)X_1^{em} + (1+t_2)X_2^{em} = (1-t_w)w$

(ii) e = 1: perfect information.

The firm's problem is simply giving the wage which is his productivity. And the government maximizing the representative individual's utility now faces the government budget constraint $N\left(t_w w + \sum_{i=1}^{2} t_i X_i^{em}\right) = \overline{G}$ only. If information is imperfect, we would need additional constraints such as incentive compatibility that appears in the imperfect information model. Here, there is no unemployment and no moral hazard due to perfect information. Workers now work the hours eL=1 specified in the given employment contract without committing moral hazard under perfect monitoring (e=1). This ideal case leads to the following standard result.

Proposition 1. *The optimal commodity tax rates are uniform.*

Proof: This is a simplified version of Atkinson and Stiglitz (1986)'s proof. Since this proof helps a more complicated discussion about the imperfect information case, we show it here. The government aims to maximize representative individual's welfare subject to the revenue constraint, labor market equilibrium condition, and the individual's conditions for utility maximization. The problem can be analyzed using the indirect utility function as suggested by Diamond and Mirrlees (1971). We define a general form of the weakly separable utility as $U(Q(X_1, X_2), \overline{T} - e)$, and substitute the indirect subutility $v(1+t_1, 1+t_2, (1-t_w)w)$ for the direct subutility of Q(.) into U(.): $U\left(v(1+t_1, 1+t_2, (1-t_w)w), \overline{T} - e\right)$. It is simply referred to as $V\left(q, (1+t_w)w, e\right)$, the indirect utility "conditional on e," where q is the price vector: $q \equiv [q_1, q_2] = [1+t_1, 1+t_2]$. Now, the government problem is:

$$L(t_1, t_2, t_w) = V\left(q, (1+t_w)w, e\right) + \lambda_1 \left(N\left(t_w w + \sum_{i=1}^{2} t_i X_i^{em} \right) - \overline{G} \right) + \lambda_2 (1-e). \qquad (11)$$

The first-order condition for t_k is:

$$\frac{\partial V\left(q, (1+t_w)w, e=1\right)}{\partial q_k} = -\lambda_1 N \left(X_k^{em} + \sum_{i=1}^{2} t_i \frac{\partial X_i^{em}}{\partial q_k} \right) \text{ for } k = \text{goods 1, 2.} \qquad (12)$$

Denoting μ as the marginal utility of income, and using the Loy's identity: $\partial V / \partial q_k = -\mu X_k^{em}$,[16] we can rearrange the above equation as follows:

$$\sum_{i=1}^{2} t_i \frac{\partial X_i^{em}}{\partial q_k} = -\frac{(\lambda_1 - \mu)}{\lambda_1} X_k^{em} \quad \text{for } k = 1, 2. \qquad (13)$$

The previous equation can be succinctly expressed using the Slutsky relationship:

$$\frac{\partial X_i^{em}}{\partial q_k} = S_{ik} - X_k^{em} \frac{\partial X_i^{em}}{\partial w} \quad \text{for all } i \text{ and } k. \qquad (14)$$

where S_{ik} is the derivative of the compensated demand curve and $\frac{\partial X_i^{em}}{\partial w}$ denotes the income

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$$\begin{aligned} \partial V^{em} / \partial q_k &= (\partial U^{em} / \partial v)(\partial v / \partial q_k) \\ &= (\partial U^{em} / \partial v)(-mu^{em} X_k^{em}), \text{ where } mu \equiv \partial v / \partial \text{income} \\ &= (\partial U^{em} / \partial v)(-mu^{em}) X_k^{em} \\ &= -\mu^{em} X_k^{em}. \end{aligned}$$

effect. Substitution of this into equation (13) yields:

$$\sum_{i=1}^{2} t_i S_{ik} = -\left(1 - \sum_{i=1}^{2} t_i \frac{\partial X_i^{em}}{\partial w} - \frac{\mu}{\lambda_1}\right) X_k^{em} \quad \text{for } k = 1, 2. \tag{15}$$

Using the symmetry of the Slutsky terms, and introducing $\theta \equiv \left(1 - \sum_{i=1}^{2} t_i \frac{\partial X_i^{em}}{\partial w} - \frac{\mu}{\lambda_1}\right)$ for the coefficient of the earlier equation, we obtain the following equation:

$$t_1 S_{11} + t_2 S_{21} = -\theta X_1^{em} \tag{16}$$

$$t_1 S_{21} + t_2 S_{22} = -\theta X_2^{em} \tag{17}$$

The sign of θ is positive because of the negative semi-definiteness of the Slutsky matrix:

$$\sum_{k=1}^{2} \sum_{i=1}^{2} t_k S_{ik} t_i = -\theta\left(\overline{G} - t_w w\right), \tag{18}$$

where the right-hand side of (18) is from the definition of budget constraint.

Solving for t_1 and t_2, we obtain the following:

$$t_1 = \frac{\theta}{S}\left(S_{12} X_2^{em} - S_{22} X_1^{em}\right) \tag{19}$$

$$t_2 = \frac{\theta}{S}\left(S_{21} X_1^{em} - S_{11} X_2^{em}\right), \tag{20}$$

where $S = S_{11} S_{22} - S_{12}^{2}$ and it is positive by the properties of the Slutsky matrix. Defining the elasticities of compensated demand as $\varepsilon_{ij} = q_j S_{ij} / X_i^{em}$, we can rearrange equations (19) and (20) as follows:

$$\frac{t_1}{1+t_1} = \frac{t_2}{1+t_2}\left(\frac{\varepsilon_{12} - \varepsilon_{22}}{\varepsilon_{21} - \varepsilon_{11}}\right). \tag{21}$$

Using the properties of the Slutsky terms, we can obtain the following well-known property in microeconomics:

$$\sum_{i=1}^{2} U_{X_i} S_{ik} = 0 \quad \text{for } k\text{= price of good 1 or 2.} \tag{22}$$

The expression can be converted to $\sum_{i=1}^{2} q_i S_{ik} = 0$ for all *k*. If we divide the expression by $X_i = 0$, it is simplified as:

$$\varepsilon_{11} + \varepsilon_{12} = 0 \quad \text{and} \quad \varepsilon_{21} + \varepsilon_{22} = 0 . \tag{23}$$

Substituting these equations into (21), the final result is the uniform commodity taxation:

$$t_1^* = t_2^* . \tag{24}$$

The uniform commodity taxation results in under any weakly separable utility if information is perfect.

Lemma 1. *There exists a set of optimal commodity and income tax rates satisfying the property of uniform commodity taxation.*

Proof: As equation (24) suggests, the uniformity of commodity tax rates holds for any income tax rate if the set of optimal commodity and income tax rates satisfy the individual and government budget constraints. Since the government budget constraint is derived directly from individual budget constraint, we check the latter one here. If the commodity tax rates are uniform, then a given optimal commodity and effort choices can be implemented by using the set of the commodity and income tax rates satisfying the following individual budget constraint:

$$\{t, t_w\} \quad \text{s.t.} \quad (1+t)X_1^{em^*} + (1+t)X_2^{em^*} = (1-t_w)w^* . \tag{25}$$

Note that since the number of equation is one and there are two variables $\{t, t_w\}$, the set of tax rates can be defined.

Lemma 2. *The optimal commodity tax rates are uniform even if there exists a constraint on the* *level of tax rate on a certain commodity.*

Proof: Suppose that we have calculated the *unconditional* optimal commodity tax rates after normalizing the budget constraint as follows. First, the generic form of the individual budget constraint $(1+t_1)X_1^{em}+(1+t_2)X_2^{em}=(1-t_w)w$ can be reexpressed as the following after dividing both sides by $(1-t_w)$:

$$\frac{(1+t_1)}{(1-t_w)}X_1^{em}+\frac{(1+t_2)}{(1-t_w)}X_2^{em}=w \;\Rightarrow\; (1+t_1{}')X_1^{em}+(1+t_2{}')X_2^{em}=w, \tag{26}$$

where $t_i{}'=(1+t_i)/(1-t_w)-1$.

Substituting the equation for *w* in (26) into the government budget constraint results in:

$$\overline{G}=N\left(t_w w+\sum_{i=1}^{2}t_i X_i^{em}\right)=N\left(\sum_{i=1}^{2}\left(t_w(1+t_i{}')+t_i\right)X_i^{em}\right)=N\left(\sum_{i=1}^{2}t_i{}'X_i^{em}\right). \tag{27}$$

Setting up the government maximization without the income tax rate, we will again obtain the unconditional optimal tax rates: $t_1{}'^{*}=t_2{}'^{*}=t^{*}$ and t_w^{*}=0.

Now, we consider the case where there is a constraint on one good, e.g., $t_1=\bar{t}$. Is $t_2=\bar{t}$ also optimal? The answer is yes. To show this, we normalize the budget constraint *conditional* on $t_1=\bar{t}$ this way.

$$(1+\bar{t})X_1^{em}+(1+t_2)X_2^{em}=(1-t_w)w \;\Leftrightarrow\; \frac{(1+\bar{t})}{(1-t_w)}X_1^{em}+\frac{(1+t_2)}{(1-t_w)}X_2^{em}=w \tag{28}$$

$$\Leftrightarrow\; (1+\tilde{t}_1)X_1^{em}+(1+\tilde{t}_2)X_2^{em}=w,$$

where $\tilde{t}_1=\dfrac{(1+\bar{t})}{(1-t_w)}-1$ and $\tilde{t}_2=\dfrac{(1+t_2)}{(1-t_w)}-1$.

Substituting the equation for *w* in (28) into the government budget constraint results in:

$$\overline{G}=N\left(t_w w+\sum_{i=1}^{2}t_i X_i^{em}\right)=N\left(\sum_{i=1}^{2}\left(t_w(1+\tilde{t}_i)+t_i\right)X_i^{em}\right)=N\left(\sum_{i=1}^{2}\tilde{t}_i X_i^{em}\right). \tag{29}$$

From this problem, we still can achieve the same $\tilde{t}_1^* = \tilde{t}_2^* = t^*$ while $t_w^* = 0$. The actual meaning is that if the statutory commodity tax rate is $t_1 = \bar{t}$, then setting $t_2 = \bar{t}$ and $t_w =$ the solution to $\overline{\mathrm{G}} = N\left(t_w w + \sum_{i=1}^{2} \bar{t} X_i^{em} \right)$ will lead to the identical optimality. In other words, the constraint on one good's tax rate does *not* function as a constraint in effect.

2. Structure of optimal commodity tax rates under imperfect information

The government problem

With the basic properties of our model, we now present the optimal taxation problem and the results below. A general form of the optimal taxation problem posited below fully considers a representative individual's commodity and effort choices { $X_1^{em}, X_2^{em}, X_1^{un}, X_2^{un}, e$ }, government budget constraint (first constraint) and the market equilibrium conditions (goods market and labor market equilibria: first, second and third constraints).

$$
\begin{aligned}
L(t_1, t_2, t_w, w, e) = {} & (1-\pi(e,d))V^{em}(q,(1-t_w)w,e) + \pi(e,d)V^{un}(q,(1-\phi t_w)b,1) \\
& + \lambda_1 \left\{ N\left[(1-\pi(e,d))\left(t_w w + \sum_{i=1}^{2} t_i X_i^{em} \right) + \pi(e,d)\left(\phi t_w b + \sum_{i=1}^{2} t_i X_i^{un} \right) \right] - \overline{G} - \pi(e,d)b \right\} \\
& + \lambda_2 (w-e) \\
& + \lambda_3 (e - e(\cdot)) \\
& + \lambda_4 (c - e_d)
\end{aligned}
$$

where $$X_1^{em} = X_1^{em}(1+t_1, 1+t_2, (1-t_w)w),\text{[17]} \qquad (30)$$

$$X_1^{un} = X_1^{un}(1+t_1, 1+t_2, (1-\phi t_w)b),$$

$$X_2^{em} = X_2^{em}(1+t_1, 1+t_2, (1-t_w)w),$$

[17] See the exact expressions for commodity demand functions in equation (3).

$X_2^{un} = X_2^{un}(1+t_1, 1+t_2, (1-\phi t_w)b)$,

$e(\cdot)$ is the effort function defined from the first-order condition for e:

$\pi_e(U^{em} - U^{un}) + (1-\pi(e,d))U^{em}_{\bar{T}\text{-}e} = 0$; $e_d = \partial e(\cdot)/\partial d$,

λ_1, λ_2 and λ_3 are the Lagrange multipliers.

Proposition 2. *Under imperfect information, the optimal commodity tax rates are uniform when the government can freely choose all commodities' tax rates.*

Proof: The first-order condition for t_k (k=1, 2) is:

$$
\begin{aligned}
&(1-\pi(e,d))\frac{\partial V^{em}\left(q,(1-t_w)w,e\right)}{\partial q_k} + \pi(e,d)\frac{\partial V^{un}\left(q,(1-\phi t_w)b,0\right)}{\partial q_k} = \\
&-\lambda_1 N\left[(1-\pi(e,d))\left(X_k^{em} + \sum_{i=1}^{2} t_i \frac{\partial X_i^{em}}{\partial q_k}\right) + \pi(e,d)\left(X_k^{un} + \sum_{i=1}^{2} t_i \frac{\partial X_i^{un}}{\partial q_k}\right)\right] \\
&-\lambda_3 \frac{\partial e(\cdot)}{\partial q_k} \\
&-\lambda_4 \frac{\partial e_d(\cdot)}{\partial q_k}.
\end{aligned}
\qquad (31)
$$

All the notations were defined earlier. At this point, it may be useful to discuss the signs and meanings of the last two terms in (31). $\frac{\partial e(\cdot)}{\partial q_k}$ is likely to be positive since the commodity tax is often referred to as more painful for low-income individuals or more simply, commodity tax is regressive. A similar prediction is expected for the sign of $\frac{\partial e_d(\cdot)}{\partial q_k}$ because the effect of monitoring may be more effective when the environment is more painful for low-income unemployed workers due to commodity taxation. λ_3 and λ_4 are negative because increases in effort and monitoring cost lower the value of Lagrange function. Denoting μ^{em} and μ^{un} as the marginal utility of income for employed workers and unemployed workers respectively, and using the Loy's identity: $\partial V^{em} / \partial q_k = -\mu^{em} X_k{}^{em}$ and $\partial V^{un} / \partial q_k = -\mu^{un} X_k{}^{un}$, we can rearrange the above equation as follows:

$$\sum_{i=1}^{2} t_i \left((1-\pi)\frac{\partial X_i{}^{em}}{\partial q_k} + \pi \frac{\partial X_i{}^{un}}{\partial q_k} \right) = -\left[(1-\pi)\frac{(\lambda_1 - \mu^{em})}{\lambda_1} X_k{}^{em} + \pi \frac{(\lambda_1 - \mu^{un})}{\lambda_1} X_k{}^{un} \right] - \frac{\lambda_3}{\lambda_1}\frac{\partial e(\cdot)}{\partial q_k} - \frac{\lambda_4}{\lambda_1}\frac{\partial e_d(\cdot)}{\partial q_k}.$$

. (32)

The previous equation can be succinctly expressed using the Slutsky relationship:

$$\frac{\partial X_i{}^{em}}{\partial q_k} = S_{ik}^{em} - X_k{}^{em} \frac{\partial X_i{}^{em}}{\partial w} \quad \text{and} \quad \frac{\partial X_i{}^{un}}{\partial q_k} = S_{ik}^{un} - X_k{}^{un} \frac{\partial X_i{}^{un}}{\partial b}$$

for all *i* and *k*.

where S_{ik}^{em} is the derivative of the compensated demand curve for an employed worker and $\frac{\partial X_i{}^{em}}{\partial w}$ denotes the income effect; S_{ik}^{un} and $\frac{\partial X_i{}^{un}}{\partial w}$ are the counterparts for an unemployed worker. Substitution of this into equation (32) yields:

$$\sum_{i=1}^{2} t_i \left((1-\pi) S_{ik}^{em} + \pi S_{ik}^{un} \right) = -\left[(1-\pi)\left(1 - \sum_{i=1}^{2} t_i \frac{\partial X_i{}^{em}}{\partial w} - \frac{\mu^{em}}{\lambda_1} \right) X_k{}^{em} + \pi \left(1 - \sum_{i=1}^{2} t_i \frac{\partial X_i{}^{un}}{\partial b} - \frac{\mu^{un}}{\lambda_1} \right) X_k{}^{un} \right] - \frac{\lambda_3}{\lambda_1}\frac{\partial e(.)}{\partial q_k} - \frac{\lambda_4}{\lambda_1}\frac{\partial e_d(.)}{\partial q_k}$$

for k= 1, 2. (33)

Using the symmetry of the Slutsky terms, and introducing θ for the coefficient of the earlier equation, we obtain the following equations:

$$t_1\left((1-\pi) S_{11}^{em} + \pi S_{11}^{un} \right) + t_2\left((1-\pi) S_{21}^{em} + \pi S_{21}^{un} \right) = -\left((1-\pi)\theta^{em} X_1{}^{em} + \pi\theta^{un} X_1{}^{un} \right) - \frac{\lambda_3}{\lambda_1}\frac{\partial e}{\partial q_1} - \frac{\lambda_4}{\lambda_1}\frac{\partial e_d}{\partial q_1} \quad (34)$$

$$t_1\left((1-\pi)S_{12}^{em}+\pi S_{12}^{un}\right)+t_2\left((1-\pi)S_{22}^{em}+\pi S_{22}^{un}\right)=-\left((1-\pi)\theta^{em}X_2^{em}+\pi\theta^{un}X_2^{un}\right) -\frac{\lambda_3}{\lambda_1}\frac{\partial e}{\partial q_2}-\frac{\lambda_4}{\lambda_1}\frac{\partial e_d}{\partial q_2} \quad . \tag{35}$$

Solving for t_1 and t_2, we obtain the following:

$$t_1=\frac{1}{S'}\left[\left((1-\pi)S_{21}^{em}+\pi S_{21}^{un}\right)\overline{X}_2'-\left((1-\pi)S_{22}^{em}+\pi S_{22}^{un}\right)\overline{X}_1'\right] \tag{36}$$

$$t_2=\frac{1}{S'}\left[\left((1-\pi)S_{12}^{em}+\pi S_{12}^{un}\right)\overline{X}_1'-\left((1-\pi)S_{11}^{em}+\pi S_{11}^{un}\right)\overline{X}_2'\right] \tag{37}$$

where $S'=\left((1-\pi)S_{11}^{em}+\pi S_{11}^{un}\right)\left((1-\pi)S_{22}^{em}+\pi S_{22}^{un}\right)-\left((1-\pi)S_{12}^{em}+\pi S_{12}^{un}\right)^2$;[18]

$$\overline{X}_i'=\left((1-\pi)\theta^{em}X_i^{em}+\pi\theta^{un}X_i^{un}+\frac{\lambda_3}{\lambda_1}\frac{\partial e}{\partial q_i}+\frac{\lambda_4}{\lambda_1}\frac{\partial e_d}{\partial q_i}\right).$$

Now, to simplify the expression above, we use the properties of the Slutsky terms and the definition of the elasticities of compensated demand as $\varepsilon_{ij}=q_jS_{ij}/X_i$.

we can rearrange equations (34) and (35) to obtain the following:

$$\frac{t_1}{1+t_1}=\frac{t_2}{1+t_2}\left(\frac{\left((1-\pi)\varepsilon_{21}^{em}X_2^{em}+\pi\varepsilon_{21}^{un}X_2^{un}\right)\overline{X}_1'-\left((1-\pi)\varepsilon_{11}^{em}X_1^{em}+\pi\varepsilon_{11}^{un}X_1^{un}\right)\overline{X}_2'}{\left((1-\pi)\varepsilon_{12}^{em}X_1^{em}+\pi\varepsilon_{12}^{un}X_1^{un}\right)\overline{X}_2'-\left((1-\pi)\varepsilon_{22}^{em}X_2^{em}+\pi\varepsilon_{22}^{un}X_2^{un}\right)\overline{X}_1'}\right). \tag{38}$$

Meanwhile, using the properties of the Slutsky terms, we can obtain the following well-known microeconomics equalities:

$$\sum_{i=1}^{2}U_{X_i}S_{ik}=0 \quad \text{for } k\text{= price of good 1 or 2.} \tag{39}$$

The expression can be converted to $\sum_{i=1}^{2}q_iS_{ik}=0$ for all *k*. If we divide the expression (39) by X_i, it is simplified as:

$$(1+t_1)S_{11}^{em}+(1+t_2)S_{12}^{em}=0 \;\Rightarrow\; \varepsilon_{11}^{em}+\varepsilon_{12}^{em}=0 \tag{40}$$

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[18] It is positive by the properties of the Slutsky matrix.

$$\left(1+t_1\right)S_{12}^{em}+\left(1+t_2\right)S_{22}^{em}=0 \;\Rightarrow\; \varepsilon_{21}^{em}+\varepsilon_{22}^{em}=0\,. \tag{41}$$

Similarly, we can define the counterparts for the unemployment state with the superscript *un*:

$$\left(1+t_1\right)S_{11}^{un}+\left(1+t_2\right)S_{12}^{un}=0 \;\Rightarrow\; \varepsilon_{11}^{un}+\varepsilon_{12}^{un}=0 \tag{42}$$

$$\left(1+t_1\right)S_{12}^{un}+\left(1+t_2\right)S_{22}^{un}=0 \;\Rightarrow\; \varepsilon_{21}^{un}+\varepsilon_{22}^{un}=0\,. \tag{43}$$

Substituting these equations into (37), it is straightforward to see that the final result is the uniform commodity taxation:

$$t_1^{\,*}=t_2^{\,*}\,. \tag{44}$$

That is, uniform commodity taxation is optimal when the government can choose commodity tax rates freely.

Meanwhile, other things being constant, it is also clear that as the terms $\partial e/\partial q_k$ and $\partial e_d/\partial q_k$ become greater positive numbers,[19] the optimal (uniform) tax rate rises as we see in equation (30). The intuition is that by increasing commodity tax rates, we can alleviate moral hazard, which boosts effort supply and efficiency at the same time. In short, the optimal commodity tax rates are affected by not only the commodity market distortions but also the effort distortion due to progressive income taxation and redistributive social welfare under imperfect information. If the effort distortion is substantial, then we would need a greater commodity tax rates as optimal solution, which in turn lowers unemployment due to a greater effort and a greater monitoring.

Proposition 3. *The optimal uniform commodity tax rate is unique.*

Proof: One may think that the uniformity of commodity tax rates holds for any level of income tax rate, but this is not true. Suppose we have calculated the optimal commodity and income tax

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[19] It was shown earlier that the usual case may satisfy: $\partial e/\partial q_i>0$, $\partial e_d/\partial q_i>0$, $\lambda_1<0$, $\lambda_3<0$ and $\lambda_4<0$. If the effort function is more sensitive to commodity taxes, then the last two terms get more negative.

rates from the government problem specified above. Then the optimal commodity and effort choices $\left\{X_1^{em^*}, X_2^{em^*}, X_1^{un^*}, X_2^{un^*}, e^* = w^* - cd^*\right\}$ should satisfy the following state-contingent individual budget constraints:

$$\{t, t, t_w\} \text{ s.t. (i) } (1+t)X_1^{em^*} + (1+t)X_2^{em^*} = (1-t_w)w^* \quad (45)$$

$$\text{(ii) } (1+t)X_1^{un^*} + (1+t)X_2^{un^*} = (1-\phi t_w)b \quad (46)$$

Note that since the number of equation are two and there are two variables $\{t, t, t_w\}$, the set of income and commodity tax rates that satisfy the two constraints is unique if $\phi \neq 1$.[20] If $\phi = 1$, then we have either a set of infinitely many solutions satisfying equations (45) and (46) if $\frac{X_1^{em^*}}{X_1^{un^*}} = \frac{X_2^{em^*}}{X_2^{un^*}} = \frac{w^*}{b}$, or none existence of solution if the condition is not met. In the presence of progressive income taxation, we thus always have the unique optimal tax rates. Suppose we deviate from $\{t, t, t_w\}$ by normalizing both equations (45) and (46) with a common factor *H* > (<)1. Then this is equivalent to taxing the income at the state of unemployment more (less) heavily compared to the pre-normalization case. In this case, the effort level changes, so does the wage since *w* = *e*-*cd*.

The case of restriction on one commodity's tax rate

As mentioned briefly in the introduction, most modern governments are constrained by redistributive politics and other external environment, e.g., foreign competition. Usually, these constraints do not allow the government to freely choose not only income tax progressivity but

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[20] Solving out for t, and t_w, we obtain $t_w = \dfrac{b - w^*(\sum_{i=1}^{2} X_i^{un^*} / \sum_{i=1}^{2} X_i^{em^*})}{\phi b - w^*(\sum_{i=1}^{2} X_i^{un^*} / \sum_{i=1}^{2} X_i^{em^*})}$ and $t = \dfrac{(1-t_w)w^*}{\sum_{i=1}^{2} X_i^{em^*}} - 1$.

also commodity tax rates on some goods that are consumed more in proportion by low-income individuals. Also, a greater openness to international trade sometimes leads to a low tax on the goods that are highly substitutable to foreign goods. Perhaps, this situation is more realistic situation than the one where the government can choose all the commodities rates freely. Then, we raise a question, *Is uniform taxation still optimal?*

We should be careful about interpreting the result of uniform taxation, however. This does not mean that commodity tax rates are uniform even if there is any restriction on a particular commodity's tax rate. For instance, necessary goods are usually taxed at low rates due to redistributive reasons. In this case, the same low taxes on other goods are not optimal since doing so means a high income tax rate to keep the government revenue neutral. The point is that the usual normalization does not work in the presence of redistributive fiscal policies of progressive income tax and UI systems. To see why more specifically, we review the following result.

Lemma 3. *In the presence of progressive income taxation, the usual normalization that eliminates the income tax rate creates differential state-contingent commodity tax rates.*

Proof: In the case where $0 \le \phi < 1$, it is possible to express the original optimal tax problem with three choice variables $\{t_1, t_2, t_w\}$ including the government budget constraint as a function of commodity tax rates only. However, the normalized commodity tax rates differ across the employment and unemployment states. To see this, we rearrange the government budget constraint.

$$\begin{aligned} \overline{G} + N\pi(e,d)b &= N\left[(1-\pi(e,d))\left(t_w w + \sum_{i=1}^{2} t_i X_i^{em}\right) + \pi(e,d)\left(\phi t_w b + \sum_{i=1}^{2} t_i X_i^{un}\right)\right] \\ &= t_w\big((1-\pi)w + \pi\phi b\big) + (1-\pi)\sum_{i=1}^{2} t_i X_i^{em} + \pi\sum_{i=1}^{2} t_i X_i^{un}. \end{aligned} \tag{47}$$

Now, the individual's state-contingent budget constraints are:

$$(1+t_1)X_1^{em} + (1+t_2)X_2^{em} = (1-t_w)w \quad \text{and} \tag{48}$$

$$(1+t_1)X_1^{un} + (1+t_2)X_2^{un} = (1-\phi t_w)b\,. \tag{49}$$

Dividing both sides of (48) and (49) by $(1-t_w)$ and $(1-\phi t_w)$, respectively, we obtain:

$$\frac{(1+t_1)}{(1-t_w)}X_1^{em} + \frac{(1+t_2)}{(1-t_w)}X_2^{em} = w \quad \Rightarrow \quad (1+t_1')X_1^{em} + (1+t_2')X_2^{em} = w \tag{50}$$

$$\frac{(1+t_1)}{(1-\phi t_w)}X_1^{un} + \frac{(1+t_2)}{(1-\phi t_w)}X_2^{un} = b \quad \Rightarrow \quad (1+t_1'')X_1^{un} + (1+t_2'')X_2^{un} = b\,, \tag{51}$$

where $t_i' = \dfrac{(1+t_i)}{(1-t_w)} - 1$ and $t_i'' = \dfrac{(1+t_i)}{(1-\phi t_w)} - 1$. If the income tax system is progressive, $0 \le \phi < 1$, then t_i' is not equal to t_i'' and in fact, t_i' can be expressed as a function of t_i'': $t_i'' = f\left(\phi, t_i'\right)$. Unlike the usual normalization, the normalization in this setting creates four commodity tax variables (two times greater in number). To deal with these state-contingent commodity tax rates, we reexpress the effort function using these state-contingent commodity tax rates.

$$e = e\left(t_1^{em}, t_2^{em}, t_1^{un}, t_2^{un}, t_w^{em}, \phi t_w^{un}\right). \tag{52}$$

Substituting (50) and (51) into (47) along with the new definition of the effort function, we obtain the new government budget constraint.

$$\begin{aligned}
&\overline{G} + N\pi\big(e(t_1,t_2,t_1,t_2,t_w,\phi t_w),d\big)b \\
&\quad = \big(1-\pi\big(e(t_1,t_2,t_1,t_2,t_w,\phi t_w),d\big)\big)\sum_{i=1}^{2}\left(t_i + t_w\left(1+t_i'\right)\right)X_i^{em} \\
&\qquad\qquad + \pi\big(e(t_1,t_2,t_1,t_2,t_w,\phi t_w),d\big)\sum_{i=1}^{2}\left(t_i + \phi t_w\left(1+t_i''\right)\right)X_i^{un} \\
&\quad = \big(1-\pi\big(e(t_1,t_2,t_1,t_2,t_w,\phi t_w),d\big)\big)\sum_{i=1}^{2}t_i' X_i^{em} + \pi\big(e(t_1,t_2,t_1,t_2,t_w,\phi t_w),d\big)\sum_{i=1}^{2}t_i'' X_i^{un}.
\end{aligned} \tag{53}$$

Since the effort function in the presence of non-zero income tax rate $e\left(t_1,t_2,t_1,t_2,t_w,\phi t_w\right)$ can be reexpressed as $e\left(t_1{}', t_2{}', t_1{}'', t_2{}'', 0, 0\right)$ under the zero income tax rate, the government budget constraint is now:

$$\begin{aligned}&=\left(1-\pi\left(e\left(t_1{}', t_2{}', t_1{}'', t_2{}'', 0,0\right), d\right)\right)\sum_{i=1}^{2} t_i{}' X_i^{em} + \pi\left(e\left(t_1{}', t_2{}', t_1{}'', t_2{}'', 0,0\right), d\right)\sum_{i=1}^{2} t_i{}'' X_i^{un} \\ &= \overline{G} + N\pi\left(e\left(t_1{}', t_2{}', t_1{}'', t_2{}'', 0,0\right), d\right) b.\end{aligned} \tag{54}$$

What this equation suggests is that even if we had only two commodity tax rates, the usual normalization that gets rid of the income tax rate creates four commodity tax rates (two commodities times two states) creates state-contingent commodity tax rates in the presence of progressive income taxation. Note that in the previous perfect information case, our normalization that gets rid of income tax rate led to only two commodity tax rates.

Proposition 4. *The optimal commodity tax rates are generally non-uniform when there exists a constraint on a commodity's tax rate.*

Proof: Unlike the previous perfect information case, a constraint on commodities' tax rates functions really as a constraint. Since the optimal is *unique* with a unique level of the uniform commodity tax rate, any other uniform commodity tax set at a specified level that is different from the optimal level results in inefficiency. In principle, a solution derived with an additional constraint cannot exceed that without it. If one commodity's tax rate is set at a certain level different than the optimal level, then imposing uniformity in commodity tax rates is in principle worse than not imposing it, which is equivalent to non-uniform commodity taxation.

IV. Optimal Commodity Taxation and Unemployment

To discuss the employment effect of commodity taxation more specifically, we introduce fairly general assumptions that are based on empirical regularities.

Assumption 1. $\frac{\partial e(t_1, t_2, t_w)}{\partial t_i} \geq 0$ for i=1, 2, and $\frac{\partial e(t_1, t_2, t_w)}{\partial t_w} < 0$.

Since it is usually said that commodity taxation is regressive or more painful for low-income unemployed people in terms of utility, we assume that increases in the commodity tax rates lead at least to a non-decrease in effort. The income tax effect is negative and consistent with intuition. In fact, all these properties hold under the CES utility function.

Assumption 2. The government budget constraint $R(t_1, t_2, t_w) = 0$ has the following properties:

$\frac{\partial R(t_1, t_2, t_w)}{\partial t} \geq 0$, where t stands for any tax rates..

What this assumption implies is that the economy is at the increasing part of the "Laffer Curve," which relates tax rates to the government revenues. This seems generally true in most modern economies with a sensible tax system.

Equilibrium unemployment

Once the government sets the income and commodity tax rates, we can define equilibrium unemployment as:

$$u(t_1, t_2, t_w) = 1 - e(t_1, t_2, t_w) \, . \qquad (55)$$

The unemployment rate here is set equal to π in our static model, but any dynamic version of the model still possesses the feature that a greater job-separation rate leads to a higher unemployment rate for a given job-matching technology.

Based on this, we can define the optimal unemployment rate, which is consistent with the optimal taxation: $u\left(t_1 = t^*, t_2 = t^*, t_w = t_w{}^*\right) = 1 - e\left(t^*, t^*, t_w{}^*\right)$. We can prove that deviations from uniform commodity taxation lower unemployment.

Proposition 5. *A transition from uniform to non-uniform commodity taxation can boost employment while reducing unemployment.*

Proof: Suppose the government is currently implementing a uniform commodity tax system $\{t_1 = \bar{t}, t_2 = \bar{t}, t_w\}$. Then such a transition from uniform to non-uniform in the form of increasing the unconstrained good's tax rate can be analyzed by the following exercise:

$$\begin{aligned} \frac{du(t_1 = \bar{t}, t_2 = \bar{t}, t_w)}{dt_i} &= -\frac{de(t_1 = \bar{t}, t_2 = \bar{t}, t_w)}{dt_i} \\ &= -\left[\frac{\partial e(t_1 = \bar{t}, t_2 = \bar{t}, t_w)}{\partial t_i} + \frac{\partial e(t_1 = \bar{t}, t_2 = \bar{t}, t_w)}{\partial t_w}\frac{\partial t_w}{\partial t_i}\right] < 0 \end{aligned} \qquad (56)$$

We know that $\partial e / \partial t_i \geq 0$, $\partial e / \partial t_w < 0$, and $\partial t_w / \partial t_i < 0$,[21] according to assumptions 1 and 2 respectively. Therefore, the non-uniform taxation can lead to a decrease in unemployment.

Proposition 6. *If the tax system adopts uniform commodity tax rates that are lower than the optimal rates, introducing non-uniform commodity taxation can achieve the "double dividend" of boosting efficiency and employment at the same time.*

Proof: If one good's tax rate t' is constrained at a lower level than the optimal level t where $t' < t$, then our model suggests that the effort level is lower than the optimal level:

$$e(t, t, \tau) > e(t', t', t'_w)\,. \qquad (57)$$

In this situation, Proposition 5 suggests that raising the unconstrained good's tax rate to t^h

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[21] To see this, we need to differentiate the government revenue function: $\partial t_w / \partial t_i = -\dfrac{\partial R / \partial t_i}{\partial R / \partial t_w} < 0$.

(hence lowering the income tax rate to $t_w{}^l$ according to assumption 2) always results in a lower unemployment:

$$t_2 \uparrow \;\Rightarrow\; e(t',t',t'_w) < e(t',t^h,t_w{}^l) \;\Rightarrow\; u(t',t',t_w) > u(t',t^h,t_w{}^l). \tag{58}$$

At the same time, according to Proposition 4, there also exists a range of unconstrained good's tax rate such that deviating from uniform taxation leads to a welfare improvement:

$$t_2 \uparrow \;\Rightarrow\; e(t',t',t'_w) < e(t',t^h,t_w{}^l) \;\Rightarrow\; EU(t',t',t'_w) < EU(t',t^h,t_w{}^l). \tag{59}$$

Proposition 7. *All the previous results are robust to changes in UI benefit formula.*

Proof: In many countries, UI benefit is viewed as a constant fraction λ of labor earning, i.e., $b = \lambda w$, where λ is often referred to as the replacement ratio. However, this change does not affect our earlier results at all. First, the uniform commodity taxation is due to the weakly separable nature of utility function, and it has nothing to do with the *UI benefit formula*. Second, the non-uniform taxation result is because of the progressive income taxation combined with the presence of assistance programs for unemployed workers, not because of a specific UI benefit formula. Third, the employment effect of a non-uniform taxation is based on empirically relevant assumptions 1 and 2, not on the UI benefit formula. Fourth, the double dividend result is based on the combination of (i) redistributive fiscal policies of progressive income taxation and UI and (ii) Assumptions 1 and 2, not on the UI benefit formula.

To illustrate the intuition behind our result, consider a situation where (i) income tax progressivity is determined by the redistributive motives of individuals, (ii) the government sets the levels of income and commodity tax rates so as to finance the expenditures for UI and public goods, and (iii) the existing commodity taxes are uniform. Given the difficulty of observing effort, a progressive income tax system combined with redistributive social insurance is equivalent to treating the consumption of unemployed workers favorably (i.e., “between-states” consumption choice distortion), which thus creates moral hazard in effort supply. Introduction of non-uniform commodity taxation can lower this “between-states” consumption choice distortion, leading to a greater effort supply. The increased effort means a lower equilibrium unemployment, and it leads to a greater *utilization* of labor and hence to a greater output. These gains can outweigh the “within-state” consumption choice distortion arising from the non-uniform taxation. This point suggests that non-uniform commodity taxation is optimal. Deviations from uniform commodity taxes can thus alleviate moral hazard that arises from the usual income tax and transfer systems, which creates an efficiency gain. All these results were also numerically verified in a simulation exercise, but we omit them for brevity. The results are available at request from the author.

Policy implications

Some useful policy implications can be drawn from the results above. First, for countries facing high unemployment primarily due to redistributive tax and social insurance (e.g., UI and other welfare programs) systems, if their commodity taxes are uniform or close to uniform and the reliance on the income tax is too high, a deviation from uniform commodity tax rates may be considered to reduce income tax rates. Also, if the existing commodity taxes are non-uniform, then they do not necessarily have to be changed into the uniform tax system since implementing uniform taxes does not necessarily improve welfare. Most countries in the world impose tariffs on imported goods. In the context of this paper, tariffs can play the welfare-improving role to some extent. This point becomes more relevant when UI benefits are high in level and are generously treated in income taxation for redistributive purposes. In fact, a favorable tax treatment of unemployment benefits is found in most countries without exception, reinforcing the practical relevance of this point. In light of this point, a more practical issue may be which commodities need to taxed more than others, which naturally provides a motivation for the second implication given below.

Second, the main conclusion of this paper lends some support to the “double dividend” hypothesis. According to the double dividend hypothesis, a transition from labor taxes to taxes on pollution-generating goods or factors of production can achieve both an improvement of the environment and a reduction in distortions arising from labor taxation.[22] While the idea of double dividend is appealing for many developed countries facing high labor taxes, Bovenberg and de Mooij (1994) showed in a traditional optimal taxation setting that it is not possible to reap the strong-form double dividend[23] when the utility function is homothetic (a special form of weakly separable utility) with respect to pollution-generating goods and other goods. If there is imperfect information about worker effort, this paper suggests that strengthening environmental taxes (especially in a revenue-neutral manner) can lead to an overall efficiency gain, even if the utility function is homothetic and the production function exhibits a constant marginal cost as in standard models.

IV. Summary and Conclusion

This paper addressed the optimal commodity taxation when involuntary unemployment arises in part from redistributive fiscal policies, such as a progressive income tax system and social insurance/welfare programs. We derived the optimal commodity taxation rule in the presence of the redistributive fiscal policies in a simplified ‘general-equilibrium’ efficiency wage

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[22] See Goulder (1995) and Bovenberg (1999), for instance.
[23] The strong-form double dividend refers to the case where both environmental and non-environmental (efficiency) gains arise from the revenue-recycling: reducing labor tax combined with strengthening environmental tax.

model with effort and commodity choices. In contrast to the conventional results by Ramsey (1927) and Atkinson and Stiglitz (1976) that consider the within-state distortion only, we show that uniform commodity taxation is optimal only when the government can choose *all* commodities' tax rates without any constraint. In a more realistic case where there is at least one constraint, non-uniform commodity taxation is optimal even under (i) the utility function that is weakly separable between goods and leisure and (ii) the constant marginal cost technology. The intuition is that a deviation from uniform commodity taxation can reduce the "between-states" distortion (moral hazard arising from a progressive income tax system and social insurance/welfare programs). This gain, combined with resulting greater effort and higher utilization of labor (i.e., lower unemployment), outweighs the "within-state" consumption choice distortions.

In light of these results, some policy implications can be drawn. First, for countries facing high unemployment due to redistributive fiscal policies (progressive income tax and generous social insurance), a deviation from uniform commodity tax rates such as imposition of tariffs may be considered to boost effort and efficiency. Especially, this point becomes more relevant when UI benefits are high in level and are generously treated in terms of income tax for redistributive purposes. Second, the main conclusion of this paper lends some support to the "double dividend" hypothesis. Although environmental taxation cannot reap the strong-form double dividend in a traditional optimal taxation setting with the perfect labor market, this paper shows that if there is imperfect information about worker effort, strengthening environmental taxes on pollution-generating goods can lead to an overall efficiency gain (i.e., the strong-form double dividend), even if the utility function is homothetic and the production exhibits a constant marginal cost as in standard models.

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