

# **Bidding Rings in Privatization**

by

**Jin Kim**

Korea Institute of Public Finance  
79-6 Karak-Dong, Songpa-Gu  
Seoul, 138-774, Korea  
j kim@ki pf. re. kr  
Telephone: 82-2-2186-2220  
Fax: 82-2-2186-2309

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Running Title: Rings

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## Abstract

This paper studies privatization problems, in which a social planner sells an indivisible object to one of the agents with private valuations (types), and explains the coexistence of auctions and consortia in practice with cooperative incentive in collusive bidding behaviors. By using the theory of optimal mechanism design with dominant strategy equilibria, we show the universal existence of bidding rings (entities that exercise collusive bidding in practice) without any restriction of independent types as in the literature of collusive bidding.

Our finding of bidding rings is for generalized Vickrey auctions, which are similar to the Vickrey auction except that the payment of the winner depends not only on the second highest bid but also on other non-winners' bids. We lay out our main result for a Vickrey auction without a reserve price from the 'mechanism design approach,' while postponing the result for a Vickrey auction with a reserve price from the 'auction design approach.'

In brief, we find that the optimal mechanism is a generalized Vickrey auction without a reserve price, that it is vulnerable to the bidding ring of a Groves scheme, and that the bidding ring can be established as a consortium to avoid collusive bidding.

Key Words: Indivisible Assets, Mechanism Design, Bidding Rings, Vickrey Auctions

JEL numbers: C72, D44, H41c

## 1. Introduction

Privatization – the transfer of indivisible productive assets or responsibilities from the public to the private sector – has been widely observed in many countries recently. <sup>(1)</sup> While it seems clear that auctioning achieves a good deal of outcome efficiency and rent maximization through competition, many observers have noted that auctions in privatization and/or procurements are vulnerable to collusive bidding, which necessarily decreases the revenue from auctions. <sup>(2)</sup> This collusive bidding behavior, conducted in ‘bidding rings,’ has been discussed by several researchers. <sup>(3)</sup> However, their predictions of collusive bidding are limited to cases where firms’ types (private valuations) are assumed to be independent.

This paper shows the existence of collusive bidding behaviors in privatization even when there is neither independence of types nor common knowledge of a common prior over types. Our finding of bidding rings is for a generalized Vickrey auction without a reserve price from the ‘mechanism design approach’ (We analyze the ‘auction design approach’ for a generalized Vickrey auction with a positive reserve price in a sister paper.) Considering the fact that in those auctions the payment of the winning bidder depends on the other bidders’ bidding, we examine ex ante cooperative incentive among agents in detail and suggest the creation of consortia to get the task of privatization delegated.

In the mechanism design approach to privatization, we investigate an optimal mechanism that satisfies outcome efficiency through dominant-strategy equilibrium behaviors without two main assumptions of the Bayesian setup in the bidding rings literature; common knowledge of a common prior and independence of types. Our choice of dominant-strategy mechanisms is based on Repullo’s (1986) conclusion that there might be a loss of generality in restricting attention to direct Bayesian mechanisms. Since our focus is on direct dominant-strategy mechanisms, as Repullo (1986) showed in his Corollary 5.2, we have no loss of generality. Furthermore, we can use the equivalence between outcome efficient dominant-strategy mechanisms and Groves mechanisms.

In the approach of mechanism design, we first show that if a social planner wants

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<sup>(1)</sup> See Krishna and Tranæs (1999) and references in it.

<sup>(2)</sup> Cramton and Schwartz (2000) and Porter and Zona (1993).

<sup>(3)</sup> Graham and Marshall (1987) and McAfee and McMillan (1992).

to design a Groves mechanism with voluntary participation and without subsidy, then it seems to be a generalized Vickrey auction without a reserve price: The agent to whom the prize of privatization is assigned would pay a price within the range of her report, the other agents would get zero payoffs, and the economic rent of privatization would be split among the social planner and the winning agent according to a predetermined payment schedule that is dependent on the other non-winners' reports. For example, the payment in the genuine Vickrey auction <sup>(4)</sup> is the second highest report.

Given any generalized Vickrey auction, we also find that agents would form a bidding ring and can extract the revenue over the minimal rent of privatization through a secret private auction (called a 'knockout'). Therefore, in an auction as the optimal mechanism, a social planner would at most obtain the minimal rent of privatization; the difference between the maximum of agents' lowest valuations and the valuation of the social planner. This powerful result of generic possibility of bidding rings both specifies the fundamental existence of cooperative incentive in privatization and implies the universal presence of collusive bidding in practices.

To prevent the formation of bidding rings during a process of privatization, we suggest the creation of a consortium to which the task of privatization is delegated at the minimal rent. It is owned by all the agents and exercises internally a genuine Vickrey auction after the realization of types. Its revenue would not be returned to agents directly. Instead, the property right ratio of the consortium to each agent is fixed at the stage of consortium installation so that each agent's expected payoff through the consortium would be the same as that through full-communication. Thus, we can say that this consortium would be an 'acceptable' mechanism by all the agents at the stage of mechanism installation.

The remainder of this paper is organized as follows. The model and preliminary results from the literature are presented in section 2. In section 3 of the mechanism design approach, we construct a generalized Vickrey auction without a reserve price as the unique optimal mechanism in privatization and examine bidding rings and consortia. Section 4 concludes with discussions.

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<sup>(4)</sup> That is, a second-price sealed-bid auction.

## 2. The Model and Preliminaries

A social planner (usually, the government) wants to allocate an indivisible private object (the prize) to a finite number of agents (firms or providers): New bands of airwaves need to be assigned; Several providers emerge for the bulk demand of a local government or a community in health care service, ambulance service, and fire emergency service; New basic discoveries need to be allotted for marketable goods because of huge R&D investment costs; Social indirect capital with big fixed costs such as highways, subways, and cables should be built. The indivisibility of the prize stems from exogenous conditions such as the lack of financial markets for issuing the equities of an incumbent public firm, regulation concerns, or the existence of well-built-up private firms for the transfer of public responsibilities.

**Environment**      The set of agents  $I = \{1, 2, \dots, n\}$  is finite with a typical element  $i \in I$ . Each agent  $i$ 's monetary valuation of the prize is private information and formulated as  $i$ 's type. Thus, we consider a private values model. Let a set of possible types be an interval  $\Theta_i \equiv [b_i, c_i]$  for all  $i \in I$ . We assume that all the agents' valuations are greater than or equal to the valuation  $v_0$  of the prize to a social planner, which is normalized to be 0; that is, for all  $i \in I$

$$(A1) \quad b_i \geq v_0 = 0.$$

Thus, privatization always increases economic welfare. (A1) excludes public-private competition. <sup>(5)</sup> Since we can treat an incumbent public firm as an agent, our results can be interpreted in a proper way to that case.

We also assume that there is no 'dominance of types' over agents in the sense that any intersection (of any pair) of type intervals is not empty. Formally, we assume that

$$(A2) \quad D \equiv \bigcap_{i \in I} (b_i, c_i) = (b, c)$$

where  $b = \max_{i \in I} b_i$  and  $c = \min_{i \in I} c_i$ . If one agent's valuation is always dominated by another agent's, there is no merit to consider that agent as a competent provider. Thus,

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<sup>(5)</sup> Also called as 'managed competition' or 'market testing' in practices.

we can omit that agent from our concern. We will call  $b$  ( $b - v_0$  generally) *the minimal rent of privatization*. Surely, a social planner expect to gain at least  $b$  from privatization in any circumstance.

The set of states is the product set of type sets;  $\Theta \equiv \prod_{i=1}^n \Theta_i$  with a typical element  $\theta = (\theta_1, \dots, \theta_n)$ . We assume that there is a common prior cumulative distribution over states. The common prior on  $\Theta$  is given by a probability measure  $F$  on the (Borel) subsets of  $\Theta$  and assumed to be mutually known to the agents. Thus, all agents and a social planner believe that uncertainty is resolved according to  $F$ . Specifically and importantly, we do not need to assume common knowledge of a common prior in the sense of Aumann (1976). Furthermore, we do not need to assume that types are independent except in section 5, which deals with that case.

The assignment decision of a social planner results in a distribution of property right ratios among agents. For example, a random assignment through a lot is a method to allocate the ownership of the prize, where the expected property right ratio of the prize is  $\frac{1}{n}$  to each agent at any state. Let  $A$  denote the set of outcomes with a typical element  $a = (a_1, \dots, a_n)$  where

$$(A3) \quad A = \{ (a_1, \dots, a_n) \in \mathfrak{R}^n \mid 1 \geq a_i \geq 0 \ \forall i \ \& \ \sum_i a_i = 1 \}.$$

The set  $A$  represents a set of ownership structures. Even though public-private partnerships are frequently observed in R&D investment industries, we eliminate those cases by assuming (A1). The ownership of the prize by one firm is a vertex of  $A$ .

We assume that agents are risk neutral. Thus, the utility of type  $\theta_i$  of agent  $i$  from an outcome  $a \in A$  and a monetary transfer  $m_i \in \mathfrak{R}$  from the social planner is  $a_i \theta_i + m_i$ . We normalize each agent's utility at the status quo as 0. We call a tuple  $\gamma = \langle I, \Theta, v_0, A, F \rangle$  a privatization problem. We assume that  $\gamma$  except  $F$  is common knowledge.  $F$  is analyzed and published to agents.

**Full-information Benchmark**      If a social planner knows the true information at each state, he can maximize the gain of privatization by giving the prize to the agent with the highest valuation. That is the case of full communication or a costless screening.

Define the maximal privatization gain at state  $\theta$  as

$$g(\theta) \equiv \max_{a \in A} \left\{ \sum_{i=1}^n a_i \theta_i \right\}. \quad (1)$$

Then,  $g(\theta) = \max_j \theta_j$  at each  $\theta$ . The expression of (1) implies an interpretation of  $g(\cdot)$  as a social choice rule. Our social choice rule would be interpreted as a Pareto efficient allocation rule and/or as an Arrowian social welfare function with equal treatment.

Denote the solution in (1) as  $a^*(\theta)$  at each  $\theta$ . Then at the stage of institutional arrangement for privatization, each agent's expected utility in full-information benchmark can be calculated as

$$V_i \equiv E[a_i^*(\theta)] \quad (2)$$

where  $E$  is the expectation operator with respect to  $F$ . We assume that each agent wants to obtain at least  $V_i$  at the stage of institutional design to agree on an institutional arrangement of privatization.

### **Mechanism**

Given an environment of privatization, a social planner needs to devise an institutional arrangement by which he can withdraw information on types and allocate the prize. Theoretically, a mechanism (or game form) consisting of a message space and an outcome function should be constructed. In any game where agents with private information use a game form, each agent with a type should have an incentive not to imitate the behavior of other types of the same agent for self-interest. In our incomplete information setup, in which there is neither common knowledge of a common prior nor independence of types, the relevant equilibrium concept is dominant-strategy equilibrium: Each agent reveals her type honestly regardless of the others' behaviors.

Our choice of the dominant-strategy equilibrium concept is supported by Repullo's (1986) result that there might be a loss of generality in restricting attention to direct Bayesian mechanisms. He was right when he said that an outcome through a truth-telling Bayesian equilibrium in a direct mechanism might not be realized through a Bayesian equilibrium in a non-direct mechanism if the truth-telling is dominated by a non-truth-telling equilibrium. Since we focus on direct dominant-strategy mechanisms, as Repullo

(1986) proved in his Corollary 5.2, we have no loss of generality in restricting our attention to direct mechanisms due to the Revelation Principle. <sup>(6)</sup>

A direct mechanism is denoted by  $(\Theta, \langle s, t \rangle)$  where  $\Theta$  is the message space and  $\langle s, t \rangle$  is an outcome function which consists of an assigning rule  $s : \Theta \rightarrow A$  and a transfer scheme  $t : \Theta \rightarrow \mathbb{R}^n$ . Given a message  $\theta' \in \Theta$ ,  $s(\theta') = (s_1(\theta'), \dots, s_n(\theta'))$  assigns property right ratios and  $t(\theta') = (t_1(\theta'), \dots, t_n(\theta'))$  designates transfers. I.e.,  $s_i(\theta')$  is the property right ratio of agent  $i$  on the prize and  $t_i(\theta')$  is the transfer to agent  $i$  from a social planner when agents' reports are  $\theta'$ . Given  $\langle s, t \rangle$ ,  $i$ 's payoff with a type  $\theta_i$  and a message  $\theta'$  is  $s_i(\theta')\theta_i + t_i(\theta')$ . We will abuse the notation  $\langle s, t \rangle$  as a mechanism.

Once a direct mechanism is installed, agents face a direct revelation game with incomplete information after each agent knows her own type. We will mainly use dominant-strategy equilibria as our equilibrium concept. For the comparison with Bayesian equilibrium, however, we are sometimes restricted to assuming that the agents' types are independent, i.e.,  $F = \prod_{i=1}^n F_i$  and  $F_{-i} = \prod_{j \neq i} F_j$  where  $F_i$  is the marginal distribution of  $F$  on  $\Theta_i$ . We do not need to assume, however, that  $F$  is common knowledge among agents and a social planner.

A mechanism  $\langle s, t \rangle$  is *dominant-strategy incentive compatible* (DSIC) if every agent has an incentive to report her own type honestly in a direct revelation game regardless of the others' reports, i.e., for all  $i$ , for all  $\theta_{-i}$ , for all  $\theta_i$ , and for all  $\theta'_i$ ,

$$s_i(\theta_i, \theta_{-i})\theta_i + t_i(\theta_i, \theta_{-i}) \geq s_i(\theta'_i, \theta_{-i})\theta_i + t_i(\theta'_i, \theta_{-i}). \quad (3)$$

A mechanism  $\langle s, t \rangle$  is said to be *outcome efficient* if it always realizes the privatization gain, that is, for all  $\theta$ ,  $\sum_{i=1}^n s_i(\theta)\theta_i = g(\theta) = \max_j \theta_j$ . By the definition of  $a^*(\cdot)$  below (1), we have  $s(\cdot) = a^*(\cdot)$  in an outcome efficient mechanism  $\langle s, t \rangle$ . A mechanism  $\langle s, t \rangle$  is a *first-best dominant-strategy mechanism* if it is both outcome efficient and dominant-strategy incentive compatible.

If the difference of social objective and individual payoff from a mechanism is type-independent for each agent, then there is no incentive to tell a lie. By using the truth-telling

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<sup>(6)</sup> Dasgupta et al. (1979) or Repullo (1986).



behaviors, a social planner obtains outcome efficiency. Given any outcome efficient mechanism  $\langle s, t \rangle$ , define its *participation charge* on  $i$  at state  $\theta$  as the maximal privatization gain minus agent  $i$ 's payoff, i.e., for all  $i$  and for all  $\theta$ ,

$$h_i(\theta) \equiv g(\theta) - s_i(\theta)\theta_i - t_i(\theta). \quad (4)$$

Then we can consider agent  $i$ 's payoff as the difference of the participation charge from the maximal privatization gain since for all  $i$  and for all  $\theta$ ,

$$s_i(\theta)\theta_i + t_i(\theta) = g(\theta) - h_i(\theta). \quad (5)$$

A mechanism  $\langle s, t \rangle$  is a *Groves mechanism* if it is outcome efficient and its participation charges are lump-sum in the sense that each agent's participation charge is independent of her type, i.e., for all  $i$ , for all  $\theta_{-i}$ , for all  $\theta'_i$ , and for all  $\theta''_i$ ,

$$h_i(\theta'_i, \theta_{-i}) = h_i(\theta''_i, \theta_{-i}). \quad (6)$$

Groves mechanisms are first-best dominant-strategy mechanisms by construction. The following theorem from the mechanism design literature concludes that in our model Groves mechanisms are the unique family of the first-best dominant-strategy mechanisms.

**Theorem 1:** In a privatization problem  $\gamma$  with assumptions (A1)-(A3), a mechanism is a first-best dominant-strategy mechanism iff it is a Groves mechanism.

**Proof:** The sufficiency of a Groves mechanism was proven in Groves and Loeb (1975) and the necessity was proven in Holmström (1979) under a convexity condition, which our model satisfies by (A3). <sup>(7)</sup> ■

Therefore, by Theorem 1, we can only look at Groves mechanisms since our concern is to find first-best dominant-strategy mechanisms in our model. Furthermore, by (5) and (6), we can express each agent's payoff from a Groves mechanism as, for all  $i$  and for all  $\theta$ ,

$$s_i(\theta)\theta_i + t_i(\theta) = g(\theta) - h_i(\theta_{-i}). \quad (7)$$

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<sup>(7)</sup> A proof appears in Jackson (2001).

Thus, the winner of the prize should pay her participation charge  $h_i(\theta_{-i})$  and a non-winner should receive a subsidy of  $g(\theta) - h_i(\theta_{-i})$  at each  $\theta$  in a Groves mechanism.

One simple example of a Groves mechanism is a mechanism with zero participation charges: A social planner gives the maximal privatization gain  $g(\theta)$  to each agent with a deficit  $(n - 1)g(\theta)$  at each  $\theta$ . If each agent receives the social gain through a zero participation charge, i.e.,  $h_i(\theta) = 0$  for all  $i$  and for all  $\theta$ , the zero-charge Groves mechanism incurs a deficit for agent  $i$  at state  $\theta$  by the amount of

$$d_i^0(\theta) \equiv g(\theta) - s_i(\theta)\theta_i. \quad (8)$$

We call it agent  $i$ 's zero-charge deficit at state  $\theta$ . It is 0 for the winner or  $g(\theta)$  for the non-winners.

### 3. Optimal Mechanism Approach

In this section, we display three economic organizations of privatization in the framework of optimal mechanism design. Based on the model and preliminaries in the previous section, we first present a unique family of auctions as an optimal mechanism in an environment of privatization. For those auctions that might be expected to appear as a public method for privatization, we then show the universal possibility of collusive bidding. Finally, we formally prove the robust existence of bidding rings and consortia.

The main feature of our analysis in this section is a scheme of monetary transfers. Intuitively, the zero-charge budget deficit in (8) would be the minimal deficit that a social planner needs to make up for with participation charges that are positive and lump-sum. The necessity of monetary transfers in an environment of privatization is proven by Dudek, Kim, and Ledyard (1995) under the assumption of type independence. In our general setup with or without type independence, we show for the completeness of presentation that there is no direct mechanism that satisfies the truth-telling, outcome efficiency, and the absence of monetary transfers. Formally, a mechanism  $\langle s, t \rangle$  is called *without transfer* if for all  $i$  and for all  $\theta$ ,

$$t_i(\theta) = 0. \quad (9)$$

**Corollary 1:** In a privatization problem  $\gamma$  with assumptions (A1)-(A3), there is no first-best dominant-strategy mechanism that is without transfer.

**Proof:** Assume that there is such a mechanism  $\langle s, t \rangle$ . Set any  $i$  and take any  $\theta_i$ . To make  $\theta$ , take  $\theta_j > 0$  from  $D$  in (A2) for any other  $j \neq i$ . From (4), the participation charge of  $i$  at  $\theta$  is  $h_i(\theta) = g(\theta) - s_i(\theta)\theta_i$ . Then  $h_i(\theta) = 0$  when  $i$  becomes the winner of the prize by her reporting  $\theta_i = c_i$ , but  $h_i(\theta) = \max_{j \neq i} \theta_j > 0$  when  $i$  is not the winner by her reporting  $\theta_i = b_i$ . Thus, the participation charge is not independent of her reporting. This is a contradiction to Theorem 1. ■

By Corollary 1, we exclude random assignments without transfers from our concerns. When the trading of the prize by the winner of random assignment to the agent with the highest valuation is possible, the bargaining would not achieve outcome efficiency by the work of Myerson and Satterthwaite (1983) and our assumption (A2).

### 3.1 Auctions and Collusive Bidding

In many cases of privatization, a social planner is allowed to extract a portion of rent from agents since the planner grants a private good to agents. Whether it is explicit such as fees and bidding payments or implicit such as efforts and lobbies, the planner and the public expect to obtain a good deal of monetary transfers from agents. However, it is unusual that there is a net subsidy to agents from a social planner in privatization. We, therefore, assume that a social planner cannot give a subsidy to any agent. Formally, a mechanism  $\langle s, t \rangle$  has *no subsidy* (NS) if there is no positive transfer from a social planner to each agent at every state, i.e., for all  $i$  and for all  $\theta$ ,

$$0 \geq t_i(\theta). \tag{10}$$

Note that this condition does not exclude the case of a taxation on agents.

An institutional arrangement represented as a game form should respect the voluntary participation of agents in game theoretic situations. Let the outside option payoff  $u_i^0(\theta)$  be 0 for all  $i$  at each state  $\theta$ . This assumption contains the case of no-resale. There are several conditions of voluntary participation according to the timing of an exit. We impose the

strongest condition: At each state no agent wants to exit from an established institutional arrangement. Formally, a mechanism  $\langle s, t \rangle$  is *ex post individually rational* (EPIR) if no agent has an incentive to drop out from a mechanism at any state, i.e., for all  $i$  and for all  $\theta$ ,

$$s_i(\theta)\theta_i + t_i(\theta) \geq u_i^0(\theta) = 0. \quad (11)$$

Dominant-strategy incentive compatible mechanisms with ex post individual rationality are consistent with the assumption of ‘complete ignorance’ in the sense that no agent needs to know the distribution of the others’ types since each agent with her private information wants to participate in the mechanism and to report her type honestly regardless of the others’ types. Thus, we can assume that only a social planner has a statistical knowledge of types and announces a common prior to agents.

The following result shows that when a social planner wants to establish a mechanism that achieves both outcome efficiency through dominant-strategy equilibrium behaviors and no subsidy with voluntary participation, then at each state all the other agents except the winner get zero payoffs and the winner pays a non-negative price to the planner.

**Theorem 2:** In a privatization problem  $\gamma$  with assumptions (A1)-(A3), if a Groves mechanism  $\langle s, t \rangle$  satisfies the conditions no subsidy and ex post individual rationality, then (i) all the other agents except the winner of the prize have zero payoffs, and (ii) the payment from the winner is nonnegative and no larger than her bidding at each state.

**Proof:** From (7) and (10),  $0 \geq g(\theta) - h_i(\theta_{-i}) - s_i(\theta)\theta_i$  for all  $i$  and for all  $\theta$ . From (7) and (11),  $g(\theta) - h_i(\theta_{-i}) \geq 0$  for all  $i$  and for all  $\theta$ . Thus, we get  $g(\theta) \geq h_i(\theta_{-i}) \geq g(\theta) - s_i(\theta)\theta_i$  for all  $i$  and for all  $\theta$ . If  $i$  is not the winner at  $\theta$ ,  $h_i(\theta_{-i}) = g(\theta)$ . Thus, by (7), the payoff of every non-winning agent is 0 at every state. If  $i$  is the winner, then the lower bound is 0. Thus,  $g(\theta) = \theta_i \geq h_i(\theta_{-i}) \geq 0$ . ■

We will interpret the mechanism in Theorem 2 as a generalized version of Vickrey auction (second-price sealed-bid auction). After the realization of state  $\theta$ , every agent submits an envelope containing a sealed bid for the prize to a social planner. The winner is the agent who bids the highest amount  $\hat{\theta} \equiv \max_i \theta_i$ . While the winner pays the second highest bidding to a social planner in the genuine Vickrey auction, the predetermined

payment schedule,  $h_i(\theta_{-i})$  of an agent  $i$ , depends on the others' reports in a generalized Vickrey auction. Furthermore, there is no monetary transfer between the social planner and the other non-winning agents. The reserve price  $r$  of the auction is irrelevant in the sense that there might not be a reserve price or the reserve price is lower than the meaningful amount  $b$ , the minimal rent of privatization in (A2).

In a generalized Vickrey auction, the economic rent of privatization  $\hat{\theta}$  would be split between a social planner and the winner of the prize. One example is surely a Vickrey auction: The rent of privatization is split between a social planner and the winner at any state according to the second highest bidding. Note that a first-price sealed-bid auction is not a generalized Vickrey auction in Theorem 2 since the payment schedule for the winner is dependent upon her report.

In a generalized Vickrey auction without a reserve price in Theorem 2, agents have a cooperative incentive to collude since the payment schedule of the winning bidder depends directly on the non-winning bidders' bidding. If agents can find the winner before the exercise of the public auction in Theorem 2, the winner has an incentive to ask the non-winners to bid lower than their valuations in the public auction. By giving the non-winners the extra gain from this unlawful collusive bidding, the winner can decrease her payment in obtaining the prize.

For example, agents would meet secretly before the exercise of the public auction in Theorem 2 and hold the same auction privately, called a knockout, to find the winner. In the public auction, the non-winning agents bid  $b$  in (A2) and the winner bids  $b + \epsilon$ , where  $\epsilon$  is a very small positive number. The following corollary shows that the inability of a generalized Vickrey auction to achieve rent maximization due to bidding rings can be extended to the cases without independence of types.

**Corollary 2:** In any generalized Vickrey auction without a reserve price in Theorem 2, there is collusive bidding by which a bidding ring can get the entire extra revenue over the minimal rent of privatization that a social planner intends to get.

**Proof:** Put a state  $\theta$ . Denote the agent with the highest valuation as  $i$  with  $\theta_i = \hat{\theta}$ . By the argument explained after Theorem 2, the winner  $i$ 's payoff is  $\theta_i - h_i(\theta_{-i})$  and the intended

revenue of the social planner is  $h_i(\theta_{-i})$  at state  $\theta$ . By the same auction of a knockout, the ring can find the winner and receive a margin  $h_i(\theta_{-i}) - b$  from the winner with  $b$  being defined in (A2). In the public auction in Theorem 2, the winner bids  $b + \epsilon$ , where  $\epsilon$  is a small positive number, and the non-winners bid  $b$ . The winner gets the prize and pays  $b$  to the social planner. Thus, the ring receives  $h_i(\theta_{-i}) - b$  the extra revenue over the minimal rent of privatization that the social planner intends to get. ■

Thus, collusive bidding in the public auction is easily constructed. The fundamental problem of a ring is, then, how to distribute spoils of collusive bidding,  $h_i(\theta_{-i}) - b$  at each  $\theta$ , among agents while holding that a knockout be a Groves mechanism.

### 3.2 Bidding Rings

Collusive bidding behaviors, known as bidding rings, have been studied by several including Graham and Marshall (1987) and McAfee and McMillan (1992).<sup>(8)</sup> They show the possibility of a bidding ring under the assumption of independent types. In this subsection we find the existence of bidding rings even when there is no restriction of independent types. Graham and Marshall (1987) reports that “[Bidding] rings exist . . . over time” and that “The benefits of ring formation are shared among members.” We therefore assume that agents have formed a bidding ring to exercise a secret private auction, called a ‘knockout.’ We also assume that the ring, as a mechanism designer, knows a common prior  $F$  and can play the role of a central broker. This illegal entity has the advantage of using monetary transfers for all agents.

Following McAfee and McMillan (1992), we assume that a successful bidding ring should overcome at least the following two obstacles when it can conceal monetary transfers indirectly such as in the form of money laundering. Firstly, a knockout should be self-enforcing in that it would satisfy incentive compatibility and ex post individual rationality. Self-enforcement is a necessary condition for a stable collusion since no member has an incentive to break down the collusive agreement either by a false report or by an exit. Secondly, a bidding ring should exploit the maximal spoils so that it would satisfy outcome

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<sup>(8)</sup> Also see Von Ungern-Sternberg (1988) and Haile and Tamer (2000).

efficiency and obtain the extra revenue over the minimal rent of privatization at every state.

Aside from the exploitation of spoils, a bidding ring should manage to distribute the spoils among agents. We assume that the ring's goal is a balanced brokerage. If the expected total transfer is zero, then the ring averagely breaks even. A mechanism  $\langle s, t \rangle$  is *ex ante budget breaking* (EABB) if

$$E\left[\sum_{i \in I} t_i(\theta)\right] = 0. \quad (12)$$

The center can have surpluses sometimes and losses at other times to distribute spoils to agents. However, averagely the ring breaks even financially.

The condition of ex post individual rationality (EPIR) in (11) in a first-best dominant strategy mechanism is equivalent to  $g(\theta) - h_i(\theta_{-i}) \geq 0$  for all  $i$ , for all  $\theta_i$ , and for all  $\theta_{-i}$ . This allows us to get a lower bound of participation charge on  $i$  as  $g(\theta) \geq h_i(\theta_{-i})$  for all  $i$ , for all  $\theta_i$ , and for all  $\theta_{-i}$ . Since the participation charge on agent  $i$  must be independent of her type, the maximal amount the planner can charge on agent  $i$  without violating  $i$ 's EPIR condition can be found by minimizing lower bounds over  $i$ 's types. That is, maximal lump-sum (participation) charges are, for all  $i$  and for all  $\theta_{-i}$ ,

$$c_i(\theta_{-i}) \equiv \min_{\theta_i} g(\theta_i, \theta_{-i}) = \max_{j \neq i} \theta_j. \quad (13)$$

If this maximal lump-sum charge is greater than the zero-charge deficit  $d_i^0(\theta)$  in (8), the planner can get a surplus while holding EPIR in a Groves mechanism. The following lemma shows that the planner can get this surplus anytime for any agent.

**Lemma 1:** For all  $i$  and for all  $\theta$ ,  $c_i(\theta_{-i}) \geq d_i^0(\theta)$ .

**Proof:** The inequality is  $\max_{j \neq i} \theta_j \geq g(\theta) - s_i(\theta)\theta_i$  by (8) and (13). If  $i$  is the winner, the righthand side is 0. Otherwise, the righthand side is  $\max_j \theta_j = \max_{j \neq i} \theta_j$ . ■

Lemma 1 holds since the outside option payoff is zero at every state. If the outside option payoff is positive, the maximal lump-sum charge might be smaller than the zero-charge deficit at some states. However, since a ring has discretion to use a surplus or a debt at each state on the condition that its expected budget is zero, what matters is the expected values of surpluses and losses over states.

By changing the state-contingent surpluses in Lemma 1 into constant transfers to agents we may transform the zero-charge Groves mechanism before (8) into a Groves mechanism which satisfies ex post individual rationality in (11) and ex ante budget breaking in (12). Denote the expected value of surpluses as

$$S_i \equiv E[c_i(\theta_{-i}) - (g(\theta) - s_i(\theta)\theta_i)] = E[c_i(\theta_{-i})] - E[g(\theta) - s_i(\theta)\theta_i] \quad (14)$$

for all  $i$ . The last expression distinguishes the expected lump-sum charges with the expected zero-sum deficits.

We assume that the ring wants to distribute this expected surpluses among the members while holding ex ante budget breaking. There are many ways to carry out ex ante budget breaking in (12) with the expected surpluses. Define the domain of spoil divisions as

$$\mathbf{M} = \{ M \equiv (M_1, \dots, M_n) \in \mathfrak{R}_+^n \mid M_i \geq 0 \ \forall i \ \& \ \sum_i M_i = \sum_i S_i \}. \quad (15)$$

The following theorem shows the universal possibility of bidding rings against to the generalized Vickrey auctions in Theorem 2 when it can conceal the monetary transfers indirectly.

**Theorem 3.1:** In a privatization problem with assumptions (A1)-(A3), there exists a Groves mechanism that satisfies ex post individual rationality (EPIR) and ex ante budget breaking (EABB). This would be used as a knockout by a bidding ring.

**Proof:** Set  $M \in \mathbf{M}$ . Define a transfer scheme  $t$  such that  $t_i(\theta) = g(\theta) - s_i(\theta)\theta_i - c_i(\theta_{-i}) + M_i$  for all  $i$  and for all  $\theta$ . Then the participation charges are lump-sum since  $h_i(\theta) = c_i(\theta_{-i}) - M_i$  is  $\theta_i$ -independent for all  $i$  and for all  $\theta$ . Thus,  $\langle s, t \rangle$  is a Groves mechanism. Since  $E[\sum_i t_i(\theta)] = \sum_i S_i - \sum_i M_i = 0$  for all  $i$ ,  $\langle s, t \rangle$  is EABB. Since  $t_i(\theta) + s_i(\theta)\theta_i = g(\theta) - c_i(\theta_{-i}) + M_i = g(\theta) - c_i(\theta_{-i}) \geq 0$  for all  $i$  and for all  $\theta$ ,  $\langle s, t \rangle$  is EPIR. ■

According to Theorem 3.1, a bidding ring works as follows: After the realization of state  $\theta$ , in the knockout in Theorem 3.1, the bidding ring can find the winner  $i$  through the truth-telling reports. The winner  $i$  pays the ring a margin  $c_i(\theta_{-i}) - b$ , where  $c_i(\theta_{-i})$  is the second highest bid. In the public auction in Theorem 2, the winner  $i$  of a knockout bids  $b + \epsilon$ , where  $\epsilon$  is a small positive number, and the non-winning members bid  $b$ . The



winner  $i$  gets the prize and pays  $b$  to the social planner. Every member of the ring receives  $M_i$  from the ring. Therefore, the payoffs are  $\hat{\theta} - c_i(\theta_{-i}) + M_i$  for the winner  $i$ ,  $M_j$  for the other member  $j$ 's,  $c_i(\theta_{-i}) - b$  for the ring, and  $b$  for the social planner.

The mechanism in Theorem 3.1 is a so-called ‘‘Groves scheme with budget breaking’’ in the literature. <sup>(9)</sup> By using this mechanism, the grand ring can extract the extra gain of privatization. If the goal of the social planner is roughly to assign the prize to the agent with the highest valuation (outcome efficiency), then a second-price sealed-bid (Vickrey) auction with a reserve price of  $b \geq r$  in Theorem 2 implements this goal with the right transfer of the prize. Then, the social planner always gets  $b$ .

One example of the bidding rings in Theorem 3.1 is the mechanism of Makowski and Mezzetti (1994), who actually deal with more general environments. <sup>(10)</sup> Set  $M_i = \frac{1}{n} \sum_i S_i$  for all  $i$  to hold the equal sharing of the spoils. This scheme has the following expected payoff for agent  $i$  at the stage of mechanism installation

$$E[v_i(s(\theta), \theta_i) + t_i(\theta)] = \frac{1}{n} \sum_{j \in I} E[v_j(s(\theta), \theta_j)] = \frac{1}{n} \sum_{j \in I} V_j \quad (16)$$

where  $V_j$  is defined in (2). That is, the payoff from the Groves scheme with budget breaking might not be equal to the expected utility defined in (2) unless all agents are identical. Thus, this ring would not be accepted by all agents at the stage of mechanism installation since there might be an agent whose expected gain from the bidding ring is lower than her expected utility in (2).

When every agent in an institutional arrangement will receive at least the expected utility in (2), the institutional arrangement is considered acceptable by all agents. Formally, a mechanism  $\langle s, t \rangle$  is *acceptable* if for all  $i$

$$E[s_i(\theta)\theta_i + t_i(\theta)] \geq V_i, \quad (17)$$

where  $V_i$  is defined in (2). Since every agent in a bidding ring has the same prior, (17) is well-defined.

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<sup>(9)</sup> See Groves (1973) and Holmström (1979).

<sup>(10)</sup> See Makowski and Mezzetti (1994) and Kosmopoulou (1999) for richer contexts.

A necessary and sufficient condition for the acceptability in (17) in that each agent has the same expected payoff from the mechanism as the expected utility in (2), turns out to be the following. <sup>(11)</sup> A mechanism  $\langle s, t \rangle$  is *ex ante budget canceling* (EABC) if for all  $i$ ,

$$E[t_i(\theta)] = 0. \tag{18}$$

Even though there is a monetary transfer from a ring to each agent, the expected transfer to each agent is zero so that the budget deficit or surplus would be canceled out.

The following theorem shows the mechanism that a bidding ring will use through budget canceling if it can conceal the monetary transfer indirectly. We claim that the following mechanism, which is one of the mechanisms in Theorem 3.1, can be used as a knockout in implicit collusive bidding when agents want to enforce acceptability. Set  $M_i = S_i$  for all  $i$  to hold the acceptable sharing of the spoils.

**Theorem 3.2:** In a privatization problem with assumptions (A1)-(A3), there exists a Groves mechanism that satisfies ex post individual rationality (EPIR) and ex ante acceptability (EAA). This mechanism would be used as an acceptable knockout by a bidding ring.

**Proof:** By Theorem 3.1, it suffices to show that  $\langle s, t \rangle$  satisfies EABC. Since  $E[t_i(\theta)] = 0$  for all  $i$ ,  $\langle s, t \rangle$  is EABC. ■

We will call this mechanism a “Groves scheme with budget canceling.” The participation charge of a Groves scheme with budget canceling consists of two parts. Just as in the Groves scheme with budget breaking discussed in Theorem 3.1, the first part,  $c_i(\theta_{-i})$  for each  $i$ , is a portion for being a Vickrey auction. The second,  $S_i$  for each  $i$ , is a portion for the sharing of the revenue by the ring formation. When all the agents are identical, our Groves scheme with budget canceling is identical to the Groves scheme with budget breaking in Makowski and Mezzetti (1994).

### 3.3 Consortia

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<sup>(11)</sup> We leave the reader to check the equivalence between (17) and (18).

The reason for the existence of a bidding ring in auctions is that even though the payment schedule in an auction is determined by non-winning bidders' bidding, there is no reward to non-winning bidders from a social planner. This exercise of determining the payment schedule brings forth a cooperative incentive among agents at the stage of mechanism installation. Bidding rings are a realization of a cooperative incentive in the form of illegal behaviors. One method to avoid a bidding ring is the (ex ante) allocation of the property right to the spoils of collusive bidding. A social planner creates a consortium of agents to legalize the cooperative incentive in bidding rings. When a cooperative incentive plays a great role among agents, an association of agents as a consortium is promoted by a social planner or by agents themselves. One famous example is a consortium of firms in R&D investment.

We now consider a consortium of agents (in privatization) by using the results in previous subsections. One important difference between an auction and a consortium is that a consortium is legally allowed to make monetary transfers to agents. At the stage of mechanism design, the social planner installs and delegates the prize to a consortium, which must be owned by all the agents and whose revenue should be restricted to the usage of its benefit. The consortium internally assigns the prize by a competitive method like auctions and its sales revenue is not transferred to the members. Denote the ownership ratios of the consortium  $p_i$  for each  $i$ . Even though the participation into a consortium is compulsory, the shares of its ownership are assumed to be determined according to the agreement among agents.

Denote the revenue of a consortium from an internal auctioning as  $C(\theta) \equiv g(\theta) - h_i(\theta_{-i})$  at state  $\theta$  with  $i$  being the winner of the internal auction. Since a social planner needs to respect each agent's participation and approval in a consortium, we allow the consortium to have the 'acceptability' in (17). When every agent in an institutional arrangement will receive at least the expected utility in (2), the institutional arrangement is considered acceptable by all agents. Formally, a mechanism  $\langle s, t \rangle$  in a consortium is *acceptable* if for all  $i$

$$E[s_i(\theta)\theta_i + t_i(\theta) + p_i C(\theta)] \geq V_i, \quad (19)$$

where  $V_i$  is defined in (2).

Thus, we assume that a social planner wants to design an ‘acceptable’ consortium that satisfies outcome efficiency through truth-telling behaviors and budget canceling with voluntary participation. Define the property right ratio of the consortium for  $i$  as  $p_i = \frac{S_i}{\sum_i S_i}$ . The following modification of Theorem 3.2 is the basis of the construction of our consortium.

**Theorem 3.3:** In a privatization problem  $\gamma$  with assumptions (A1)-(A3), there exists a Groves mechanism that satisfies ex post individual rationality (EPIR) and ex ante acceptability (EAA). This mechanism would be used as an acceptable consortium by a social planner.

**Proof:** Define a transfer scheme  $t$  such that  $t_i(\theta) = g(\theta) - s_i(\theta)\theta_i - c_i(\theta_{-i})$  for all  $i$  and for all  $\theta$ . Then the participation charges are lump-sum since  $h_i(\theta) = c_i(\theta_{-i})$  is  $\theta_i$ -independent for all  $i$  and for all  $\theta$ . Thus,  $\langle s, t \rangle$  is a Vickrey auction. Since  $E[s_i(\theta)\theta_i + t_i(\theta) + p_i C(\theta)] = E[g(\theta) - c_i(\theta_{-i})] + p_i E[C(\theta)] = E[g(\theta) - c_i(\theta_{-i})] + S_i = E[s_i(\theta)\theta_i] = V_i$  for all  $i$ ,  $\langle s, t \rangle$  is acceptable. Since  $t_i(\theta) + s_i(\theta)\theta_i = g(\theta) - c_i(\theta_{-i}) \geq 0$  for all  $i$  and for all  $\theta$ ,  $\langle s, t \rangle$  is EPIR. ■

The working of an acceptable consortium in Theorem 3.3 is as follows. At the stage of consortium installation, the shares of the ownership of a consortium are determined as  $\frac{S_i}{nS}$  for each  $i$ . The social planner delegates the allocation of the prize to the consortium with the lease price of  $b$ . The consortium internally exercise a Vickrey auction and divide the rent of privatization among the winning bidder, the consortium, and the social planner. At each state, a Vickrey auction is implemented in a consortium to assign the prize to the agent with the highest valuation. The winner pays the second highest bid to the consortium and gets the prize. The consortium pays the minimal rent of privatization  $b$  to the social planner at every state. The consortium leases the prize from a social planner.

Since each agent considers the winner’s portion of the privatization rent as well as the portion of a consortium, there is no incentive to form a bidding ring. Agent  $i$  has the expected revenue from the consortium by  $S_i$  and the expected payoff from the Vickrey auction by  $E[g(\theta) - c_i(\theta_{-i})]$ . Furthermore, since the sum of the expected payoff from the consortium and the auction is the same as the expected utility in (2) from full communi-

cation, each agent has an incentive to approve the installation of the consortium.

Another feature of a consortium is that even though the revenue of a consortium can be used by itself, the social planner can control the direction of the consortium's revenue-spending. A typical method to do this is a joint consortium between the public and private sectors. The public share of a consortium is  $\frac{b}{b+nS}$  and each agent  $i$ 's share is  $\frac{S_i}{b+nS}$ .

Our legalization of the Groves bidding ring with budget canceling in Theorem 3.2 by creating the consortium in Theorem 3.3 does not depend on the assumption of independent types. Thus, the consortium can be widely used for any distribution  $F$ . Furthermore, the common prior  $F$  only needs to be mutually known to the agents. The social planner has statistical information about the types and publishes it to agents. In examples such as agricultural marketing orders, this informational assumption is not strong.

## 4. Discussions

This paper contributes to the bidding ring literature by finding the general possibility of bidding rings in optimal mechanisms in a general environment of privatization, where there is neither common knowledge of a common prior nor independent types. This analysis can be applied into procurement contracts with fixed-cost uncertainty when we consider the valuation of procurement as the consumer surplus minus a private fixed-cost.

We studied only the grand ring in this paper. We may argue that if agents know the true state through a costless detection technology, then there is no way to form a sub-ring in the grand ring. Since there is equivalence between dominant-strategy truthful implementation and Nash truthful implementation through direct mechanisms, this argument is valid if we assume common knowledge of a common prior. The endogenous formation of the stable ring and the competition among rings are beyond this paper. Our interest is in the ex ante cooperative incentive that comes from the payment schedule in optimal auctions. Since the study of cooperative game solutions in games with incomplete information has developed, it is interesting to examine the endogenous formation of the stable ring based on those solutions.

Our construction of the optimal auction cum bidding rings has generalized Vickry auctions as a domain of auctions. We claim that the same analysis of ex ante spoils could

be extended to other standard auctions. Our solution of consortia against collusive bidding is based on the commitment of a social planner to keeping the ownership of agents with fixed property right ratios. Those ratios depend on environmental factors such as the range of individual types, the distribution of types, and the number of agents. It is very important question how to make these factors endogenous.

Our understanding of a consortium - whether it is an affiliation, association, or union - as a method to avoid collusive bidding in privatization gives a balanced outlook on cooperative incentive as well as on strategic competition in privatization and procurement.

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